



How to break EW symmetry naturally?

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ITP-CAS

C. Csaki, T. Ma, [J. Shu.](#), Phys.Rev.Lett. 119 (2017) no13, 131803

C. Csaki, T. Ma, [J. Shu.](#), Phys.Rev.Lett. 121 (2018) no23, 231801

Many many other
papers.....

C. Csaki, T. Ma, [J. Shu](#), J-H. Yu, arxiv:1810.07704

C. Csaki, F. Freitas, L. Huang, T. Ma, [J. Shu](#), M. Perelstein, arxiv:1811.01961

Outline

- Basic introduction
- Brief background knowledge on Maximally Symmetric Composite Higgs
- How to realize maximal symmetry, even from warped extra dimensions (emergence!!!)
- Naturalness sum rule, how to test?
- Trigonometric Parity for the Composite Higgs
- Outlook on HEP and other fields

High energy physics

TeV

GeV

MeV

eV

t

W, Z h 2012

b
c
τ
μ
u, d

p, n

nucleus

12 orders of
magnitude

e

ν_3

ν_2

ν_1

atoms

1895

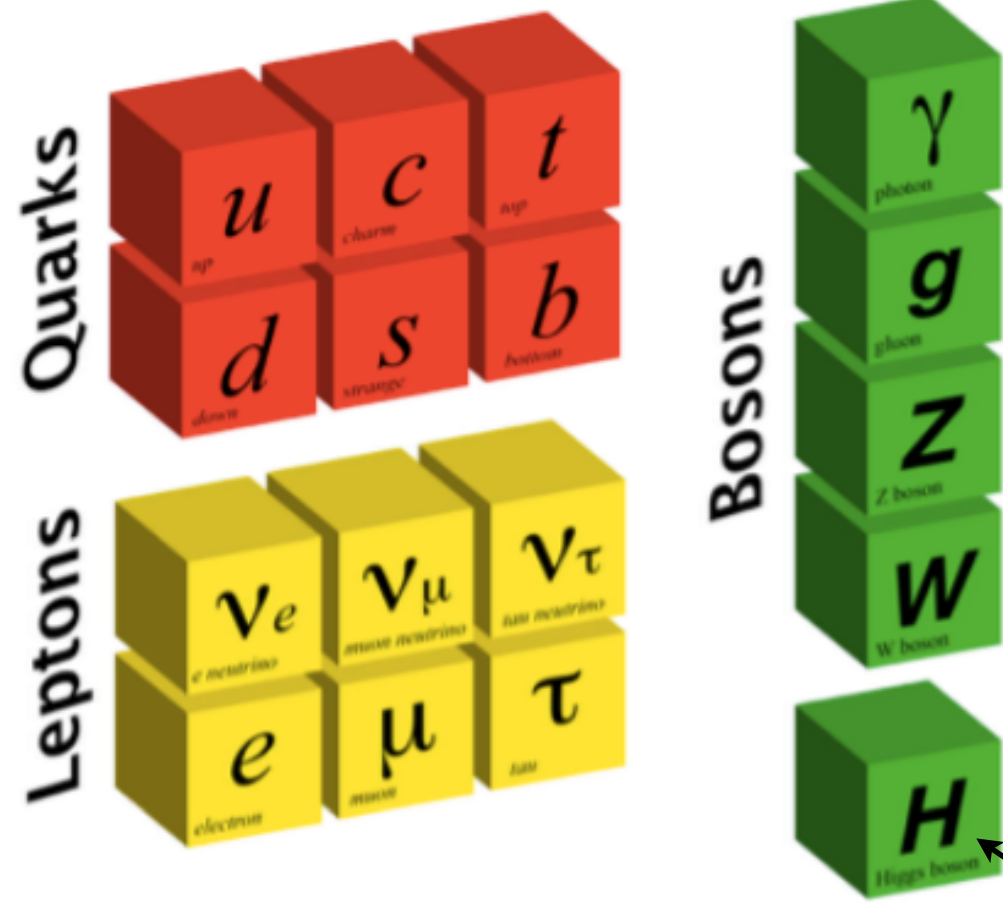
Open the door of
sub-atom physics

Higher and
higher energy

Last 122 years



“Old” physics up to date



The Weinberg-Salam Model

$$\mathcal{L} = \bar{E}_L(i\not{\partial})E_L + \bar{e}_R(i\not{\partial})e_R + \bar{Q}_L(i\not{\partial})Q_L + \bar{u}_R(i\not{\partial})u_R + \bar{d}_R(i\not{\partial})d_R + g(W_\mu^+ J_W^{\mu+} + W_\mu^- J_W^{\mu-} + Z_\mu^0 J_Z^\mu) + eA_\mu J_{EM}^\mu,$$

$$J_W^{\mu+} = \frac{1}{\sqrt{2}}(\bar{\nu}_L \gamma^\mu e_L + \bar{u}_L \gamma^\mu d_L);$$

$$J_W^{\mu-} = \frac{1}{\sqrt{2}}(\bar{e}_L \gamma^\mu \nu_L + \bar{d}_L \gamma^\mu u_L);$$

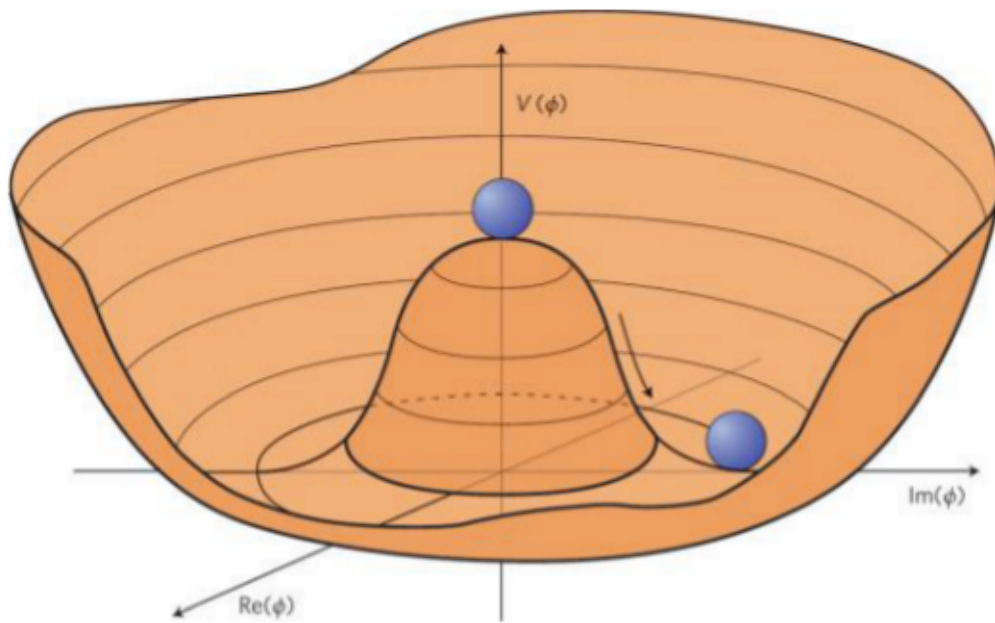
$$e = \frac{gg'}{\sqrt{g^2 + g'^2}},$$

$$J_Z^\mu = \frac{1}{\cos\theta_w} \left[\bar{\nu}_L \gamma^\mu \left(\frac{1}{2}\right) \nu_L + \bar{e}_L \gamma^\mu \left(-\frac{1}{2} + \sin^2\theta_w\right) e_L + \bar{e}_R \gamma^\mu \left(-\frac{1}{2} + \sin^2\theta_w\right) e_R + \bar{u}_L \gamma^\mu \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_w\right) u_L + \bar{u}_R \gamma^\mu \left(-\frac{2}{3}\sin^2\theta_w\right) u_R + \bar{d}_L \gamma^\mu \left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_w\right) d_L + \bar{d}_R \gamma^\mu \left(\frac{1}{3}\sin^2\theta_w\right) d_R \right]$$

$$J_{EM}^\mu = \bar{e} \gamma^\mu (-1) e + \bar{u} \gamma^\mu \left(+\frac{2}{3}\right) u + \bar{d} \gamma^\mu \left(-\frac{1}{3}\right) d.$$

The chosen one!

Why God's particle?



Gives all particles
mass

Higgs potential

$$V(h) = \frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4}h^4$$

EW
SB
(Higgs mechanism)

$$\langle h \rangle \equiv v \neq 0 \rightarrow m_W = g_W \frac{v}{2}$$

The origin of the
mass

Unknown in “old” physics

Higgs potential

$$V(h) = \frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4}h^4$$

Laudau-Ginzberg potential (Superconductivity)

$$m_h^2(h^\dagger h) + \frac{1}{2}\lambda(h^\dagger h)^2 + \frac{1}{3!\Lambda^2}(h^\dagger h)^3$$

negative, why?

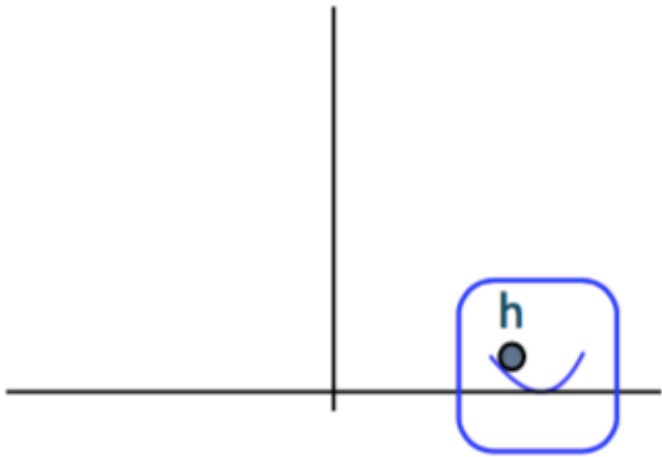
$$\frac{1}{2}\lambda(h^\dagger h)^2 \log \left[\frac{(h^\dagger h)}{m^2} \right]$$

$$V(h) \simeq -\gamma s_h^2 + \beta s_h^4.$$

We actually never know the Higgs potential and why EWSB?

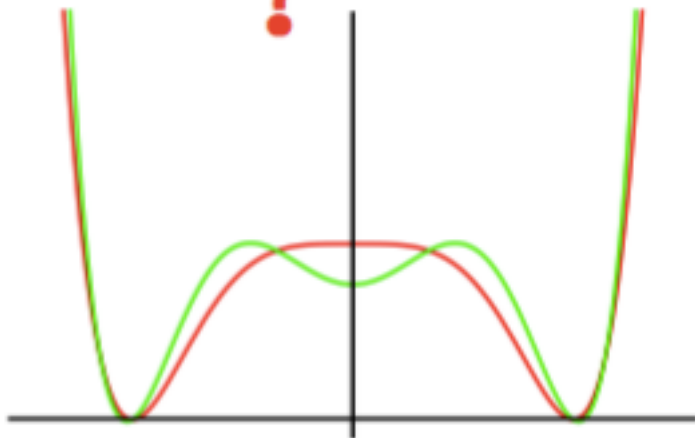
CORE question in particle physics

Unknown in “old” physics



What we know now

?



Taylor expand on the quantum fluctuation of higgs potential

$$h^3 \quad h^4 \quad h^5 \circ \circ \circ \circ h^9$$

Future Collider

Not known how to probe

Same collider signal, different potential

$$V(h) = \frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4}h^4$$

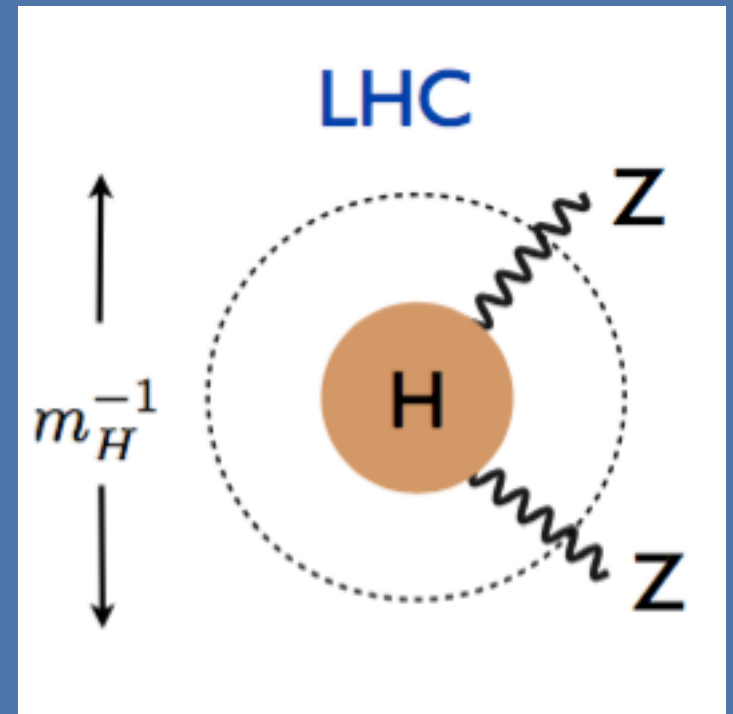
$$V(h) = \frac{1}{2}\mu^2 h^2 - \frac{\lambda}{4}h^4 + \frac{1}{\Lambda^2}h^6$$

Substructure of Higgs?

Suppose in NP scale, we see
substructure of Higgs (like
QCD Pi form factor deviations)

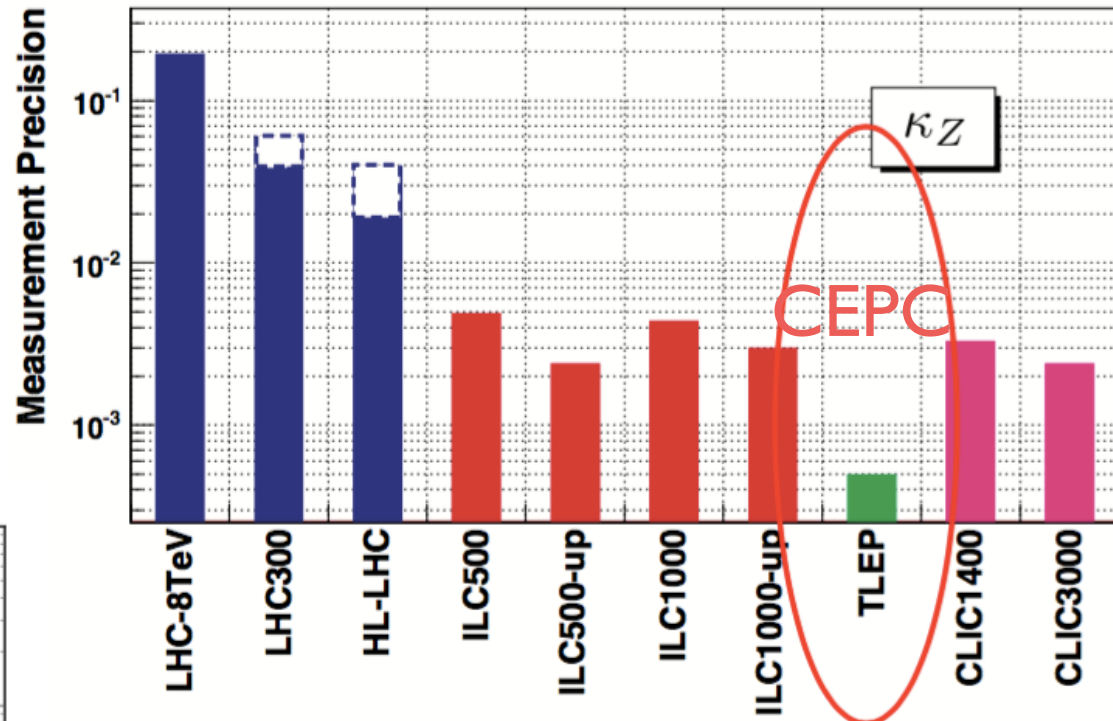
Possible NP deviation

$$\delta = c \frac{m_W^2}{M_{\text{NP}}^2}, \quad c = \mathcal{O}(1)$$

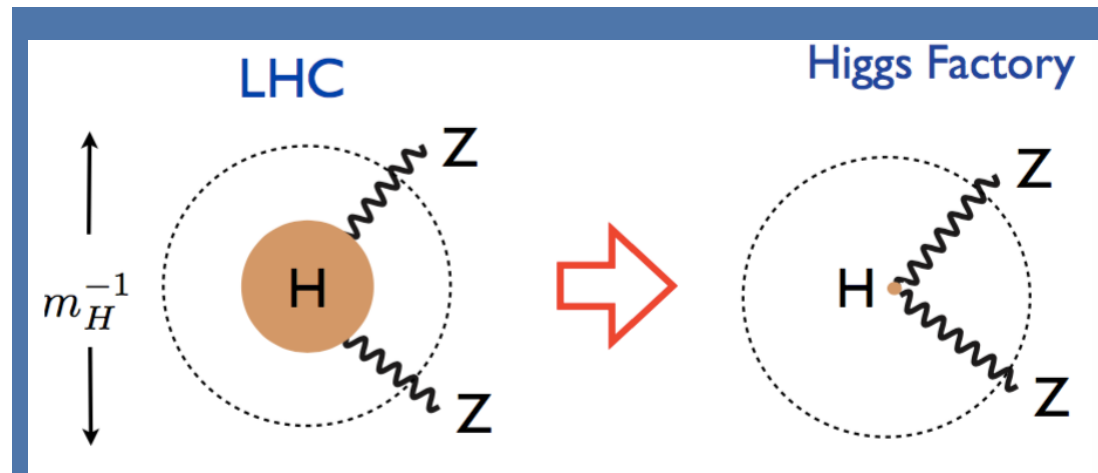
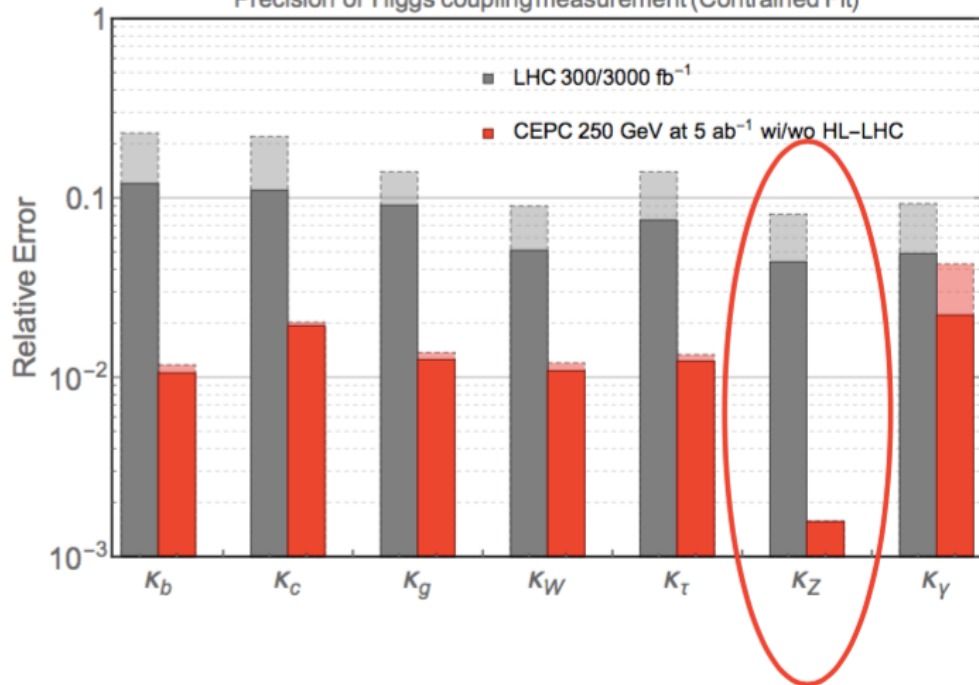


Precision Higgs measure

$$\kappa_Z = \frac{g_{hZ}(\text{Measured})}{g_{hZ}(\text{SM})}$$



Precision of Higgs coupling measurement (Constrained Fit)



Higgs compositeness for Higgs factory

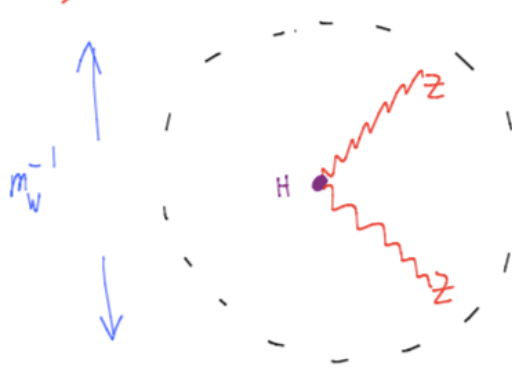


Our chairman Mao says all particles are composite from his philosophy!
A great eye-catching and physics motivation for Higgs factory and great collider
Nima's talk in IHEP 2018

Higgs precision

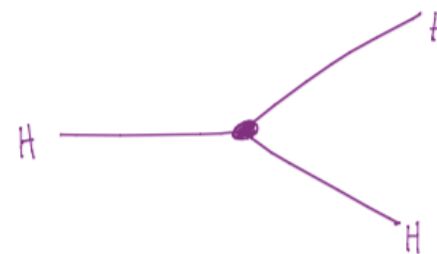
Self-interaction

Never Seen Pion-Like Scalar



Higgs Factory
+
We will know
FOR SURE
if it's "like a Pion"

Never Seen Self-Interacting Fundamental Particles



100 TeV Collider
Measured to $\sim 5\%$

Higgs as a pNGB

Why Higgs as a pNGB?

Kaplan, H. Georgi, Phys.Lett.B 136 (1984) 183

Kaplan, H. Georgi, Phys.Lett.B 145 (1984) 216

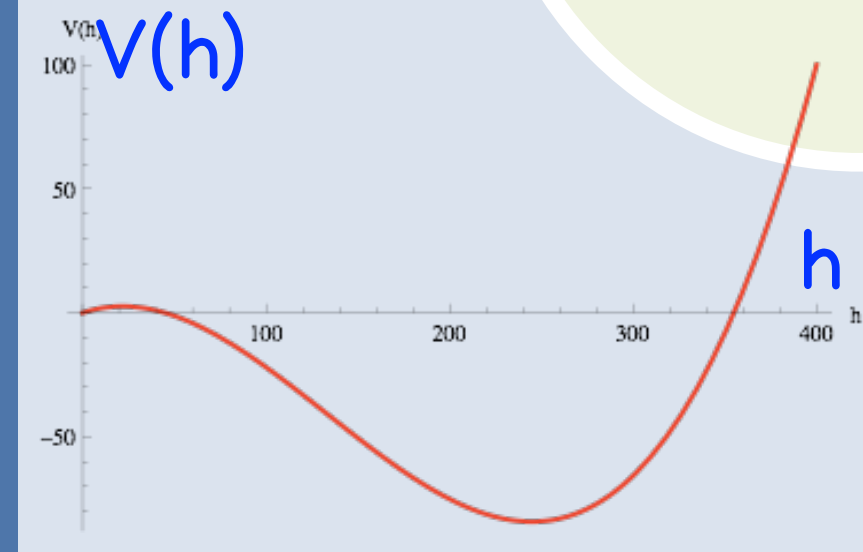
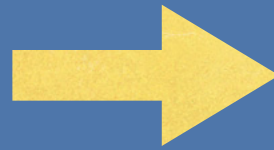
- Higgs mass small comping to confinement scale ($1 \sim 10 \text{TeV}$) **Highly constrained by LEP**
- The radiatively generated Higgs potential
- universal prediction on Higgs couplings (Like pion soft theorem)

If the **strong dynamics** triggers the breaking G/H ,
pNGB is a **composite** particle.

QCD chiral symmetry, pion

The origin of Higgs potential

But pions has no vev
only a positive mass



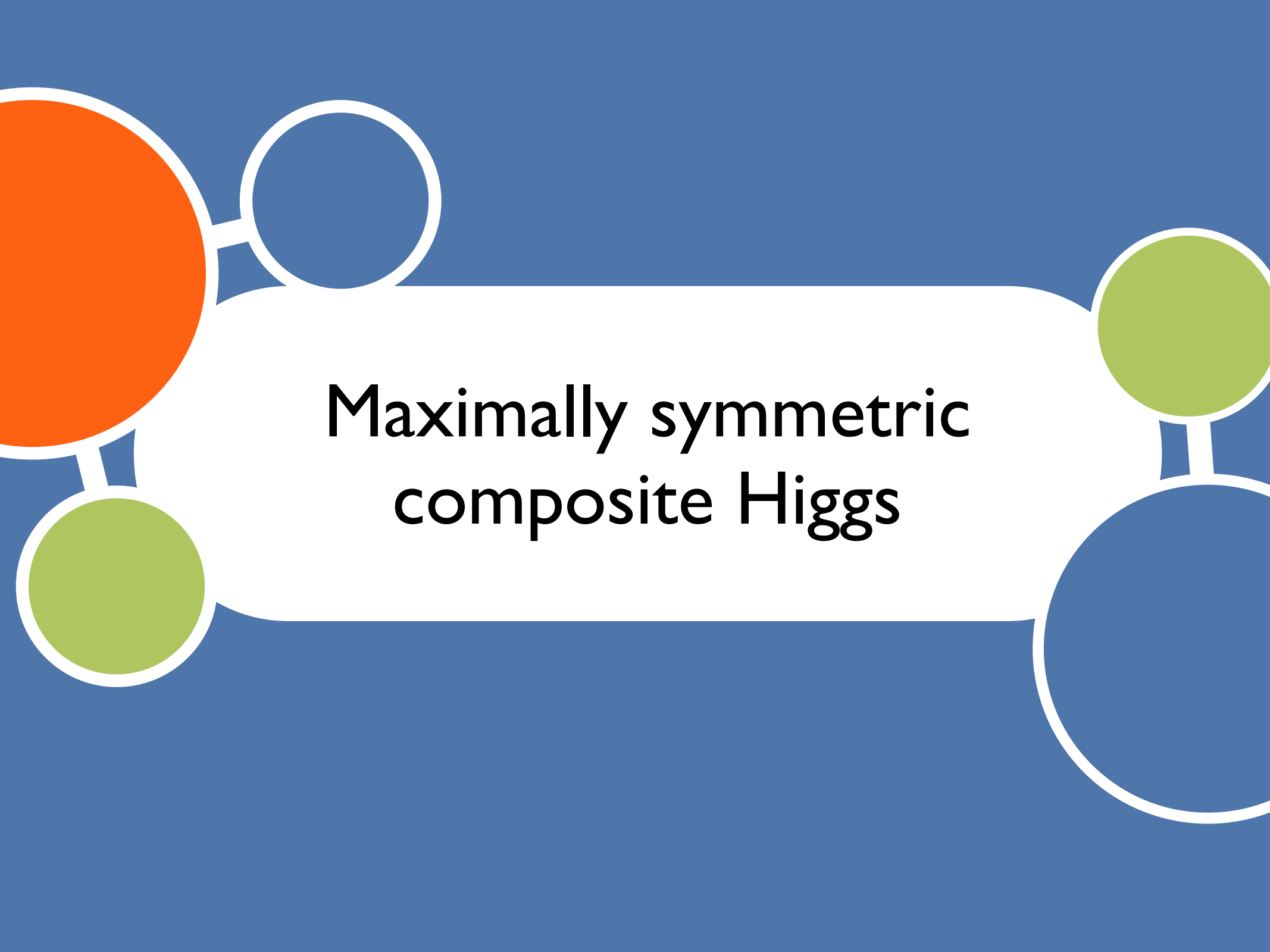
The origin of the Higgs potential

- The “parton” mass for the composite Higgs

L. Da, T. Ma, J. Shu, in preparation

- Quantum corrections from the SM particles (Mostly top)?

More focused top sector

The background is a solid blue color. In the center, there is a white, rounded rectangular text box. Surrounding this box are several decorative elements: a large orange circle on the left, a smaller white circle with a blue outline above it, a green circle below the orange one, a green circle on the right, and a large blue circle with a white outline at the bottom right. All these circles are connected to the central white box by thin white lines.

Maximally symmetric composite Higgs

Why another CHM?

- ● ● Can get the correct EWSB.
 - Can easily get the 125 GeV light Higgs mass?
 - No UV dependence of Higgs potential.
 - Can we have minimal technical tuning gauge hierarchy problem
- A **general** methods based on symmetric coset space to describe the EWSB (Higgs as a pNGB) in an **unified** manner.
 - Find the new symmetry breaking pattern (Maximal symmetry) **automatically solves all problems** above

Simplest Structure

Symmetric space

For any global symmetry G breaks into H

$T^{\hat{a}}(T^a)$ is the (un)broken generator

$$[T^a, T^a] \sim T^a, [T^a, T^{\hat{a}}] \sim T^{\hat{a}}$$

$$[T^{\hat{a}}, T^{\hat{a}}] \sim T^a$$

H is a closed group

symmetric coset space

G always has an **automorphism**

$$VT^aV^\dagger = T^a$$

$$VT^{\hat{a}}V^\dagger = -T^{\hat{a}}$$

Higgs is the NGB in the symmetric coset space

V is also the Higgs parity operator, like pion parity in QCD

Examples:

Examples of Symmetric Coset Space:

$$SU(M+N)/SU(M) \times SU(N) \times U(1)$$

$$SO(M+N)/SO(M) \times SO(N)$$

$$V = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & -1 & \\ & & & & -1 \end{pmatrix}$$

$$SO(5)/SO(4)$$

$$V = \begin{pmatrix} \mathbf{1}_{4 \times 4} & 0 \\ 0 & -1 \end{pmatrix}$$

$$SU(2N)/Sp(2N)$$

$$\Phi = \mathbf{1}_{N \times N} \times (i\sigma_2)$$

$$SU(N)/SO(N)$$

$$\Phi = \begin{pmatrix} 0 & \mathbf{1}_{\frac{N}{2} \times \frac{N}{2}} \\ \mathbf{1}_{\frac{N}{2} \times \frac{N}{2}} & 0 \end{pmatrix} \quad N = 2l$$

$$G \times G/G_V$$

$$\Phi = \begin{pmatrix} 0 & & \mathbf{1}_{\frac{N-1}{2} \times \frac{N-1}{2}} \\ 0 & 1 & 0 \\ \mathbf{1}_{\frac{N-1}{2} \times \frac{N-1}{2}} & 0 & 0 \end{pmatrix} \quad N = 2l + 1$$

Actually
cover
almost all
useful
cosets

Goldstone in symmetric space



For any global symmetry G
spontaneously breaks into H

If decouple the “Higgs”

$$U = \exp\left(\frac{ih^{\hat{a}}T^{\hat{a}}}{f}\right)$$

The CCWZ transformation

$$U \rightarrow gU h(h^{\hat{a}}, g)^{\dagger}$$

For any symmetric coset space

Key
construction

$$\Sigma' = U^2 V$$

Information of G/H
is included in V

Simple linear transformation

$$\tilde{U} = VUV^{\dagger} = U^{\dagger}.$$

$$\Sigma' \rightarrow g\Sigma'g^{\dagger}.$$

Goldstone matrix transform **linearly!**

G/H CW potential from top

Consider the MCHM5 SO(5)/SO(4)

SM Fermion

Embed the SM fields into fund rep of G (spurionic)

$$\Psi_{q_L} = \frac{1}{\sqrt{2}} \begin{pmatrix} b_L \\ -ib_L \\ t_L \\ it_L \\ 0 \end{pmatrix} \quad \Psi_{t_R} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ t_R \end{pmatrix}$$

$$\Sigma' \rightarrow g \Sigma' g^\dagger .$$

$$\Lambda^L = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 & -i & 0 \\ 1 & i & 0 & 0 & 0 \end{pmatrix} \text{ spurion vev} \quad \Lambda^R = (0 \ 0 \ 0 \ 0 \ 0 \ 1)$$

$$\Psi_{Q_L} = \Lambda_L^\alpha Q_L^\alpha$$

$$\Psi_{t_R} = \Lambda_R t_R$$

$$\Sigma' = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & c_{2h} & -s_{2h} \\ 0 & 0 & 0 & -s_{2h} & -c_{2h} \end{pmatrix}$$

Q_L^α, t_R SU(2) multiplet

G/H CW potential from top

Most general Lagrangian
Master formular

$$\Sigma'^2 = 1$$

$$\Sigma' \rightarrow g \Sigma' g^\dagger .$$

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \bar{\Psi}_{Q_L} \not{p} (\Pi_0^q(p) + \Pi_1^q(p) \Sigma') \Psi_{Q_L} \\ & + \bar{\Psi}_{t_R} \not{p} (\Pi_0^t(p) + \Pi_1^t(p) \Sigma') \Psi_{t_R} \\ & + \bar{\Psi}_{Q_L} M_1^t(p) \Sigma' \Psi_{t_R} + h.c. \end{aligned}$$

$$\Psi \rightarrow g \Psi \quad \text{fund rep}$$

Derivatives of GB does not
contribute to Higgs potential

Converting back
to the SM fields by
using spurions.

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \bar{Q}_L^\alpha \not{p} \text{Tr}[(\Pi_0^q + \Pi_1^q \Sigma') P_l^{\alpha\beta}] Q_L^\beta \\ & + \bar{t}_R \not{p} \text{Tr}[(\Pi_0^t + \Pi_1^t \Sigma') P_r] t_R \\ & + M_1^t \bar{Q}_L^\alpha t_R \text{Tr}[\Sigma' \cdot P_{lr}^\alpha] , \end{aligned}$$

$$P_l^{\alpha\beta} = (\Lambda_L^\beta)^\dagger \Lambda_L^\alpha, \quad P_r = (\Lambda_R)^\dagger \Lambda_R, \quad P_{lr}^\alpha = (\Lambda_R)^\dagger \Lambda_L^\alpha.$$

Master formular

Based on G/H, one can write whatever Lag
contribute to Higgs potential

Enlarged Global Symmetry

$\Pi_1^{q,t} = 0$ LH & RH top each has $G_L \times G_R$ symmetry

$$G_L \quad \Psi_{Q_L} \rightarrow g_L \Psi_{Q_L}$$

$$G_L \quad \Psi_{t_R} \rightarrow g_R \Psi_{t_R}$$

global

Only acting on fermions, not Goldstones

Only the mass term breaks them into $G_{V'}$

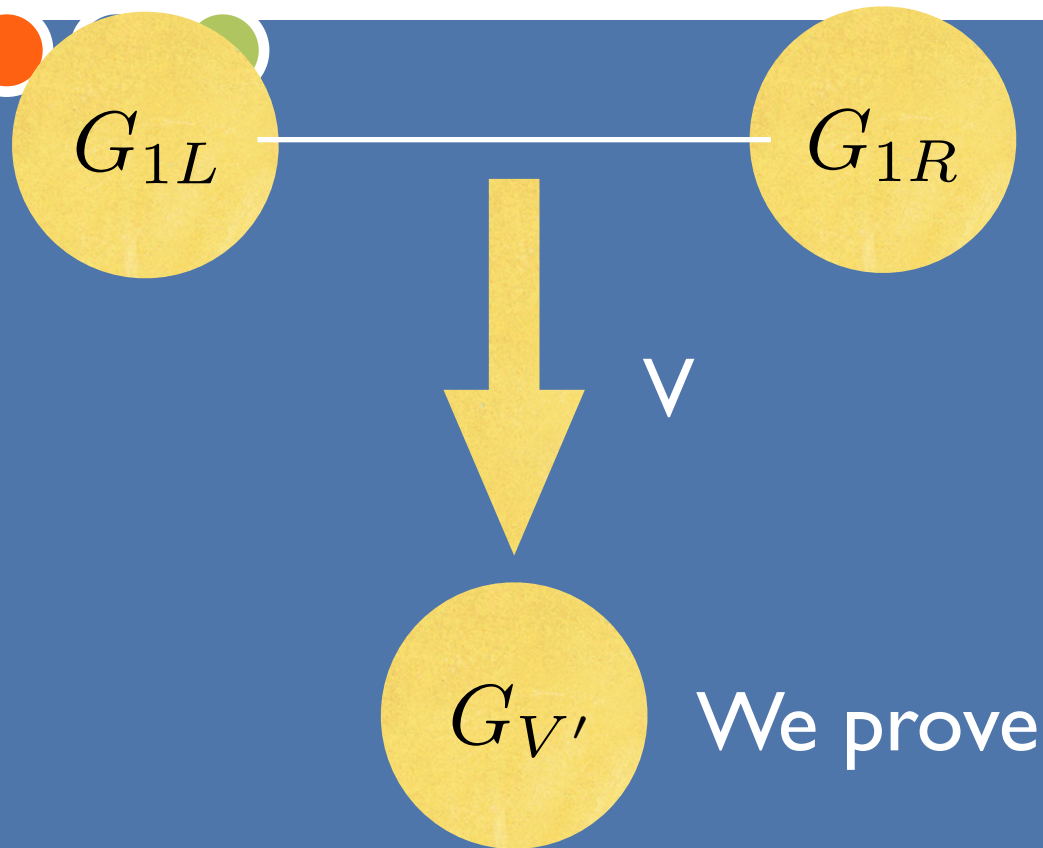
$g_L \Sigma' g_R^\dagger = \Sigma'$. Maximal subgroup leaves the GB invariant

$$g'_{L,R} = U^\dagger g_{L,R} U$$

$$g'_L V g'^R{}^\dagger = V$$

Global Symmetry from composites

What is Maximal Symmetry?



Can be used as a new UV completion of maximal symmetry just like moose model for LH,

$SO(5)/SO(4)$

$$V = \begin{pmatrix} \mathbf{1}_{4 \times 4} & 0 \\ 0 & -1 \end{pmatrix}$$

H_V

$(G/H)_A$

Mathematical structure

C. Csaki, T. Ma, [J. Shu.](#), Phys.Rev.Lett. 119 (2017) no13, 131803

Appendix A

The Higgs potential

$$V(h) = -2N_c \int \frac{d^4p}{2\pi^4} \log \left[1 + \frac{(M_1^t)^2 |\text{Tr}[\Sigma' \cdot P_{lr}^1]|^2}{p^2 \text{Tr}[\Pi_0^q P_l^{11}] \text{Tr}[\Pi_0^t P_r]} \right]$$

$\sin(2h/f)$

Higgs potential is just the top mass square up to some factors after integration on momentum.

Log(1+x) expansion

CW
potential

top mass from both LH & RH top mixture with top partners

No UV Divergence

$$M_1^t \sim \lambda_L \lambda_R f^2 (M_Q - M_S) / p^2$$

$$V(h) \sim \lambda_L^2 \lambda_R^2 f^4 (M_Q - M_S)^2 / \Lambda^2$$

$$m_t \sim \sin 2h$$

Higgs potential $(\sin 2h)^2 = s_h^2 - s_h^4$.

Understand deconstruction

Collective Symmetry Breaking, “Little Higgs”

$$\sim \int d^4p \Pi_1^{q,t}$$

$$\Pi_1^{q,t} \propto p^{-2N} \text{ at UV}$$



N=3 for finite potential

Even for large N, **still**
have very troublesome
finite piece

Higgs potential tends to have
large $v/f \gg 1$ (double tuning)

Higgs mass tends to be large
than 300GeV

Fine-tuning

Realization from MCHM5



Fermion mass from linear mixing

$$\mathcal{L}_{mix} = \lambda \bar{q}_i \mathcal{O}_i$$

$$\mathcal{O}_i \sim U \Psi_i$$

partial compositeness

$$\mathcal{L} = \lambda_L \bar{q}_L^\alpha \Lambda_{\alpha I}^L \mathcal{O}_R^I + \lambda_R \bar{t}_R \Lambda_{\alpha I}^R \mathcal{O}_L^I + h.c$$

$$U \rightarrow gU h(h^{\hat{a}}, g)^\dagger$$

Fermionic Lagrangian:

Top and top partner masses:

$$m_t = \frac{\epsilon_{qQ} \epsilon_{tS} f^2}{2M_T M_{T_1}} \left| \frac{\epsilon_{qS}}{\epsilon_{qQ}} M_Q - \frac{\epsilon_{tQ}}{\epsilon_{tS}} M_S \right| \sin \frac{\langle h \rangle}{f}$$

$$M_T = \sqrt{\epsilon_{qQ}^2 f^2 + M_Q^2}, \quad M_{T_1} = \sqrt{\frac{\epsilon_{tS}^2}{2} f^2 + M_S^2}.$$

5=4+1 Composite top partners

$$\Psi_Q = \frac{1}{\sqrt{2}} \begin{pmatrix} iB - iX_{5/3} \\ B + X_{5/3} \\ iT + iX_{2/3} \\ -T + X_{2/3} \\ 0 \end{pmatrix} \quad \Psi_S = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ T_1 \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_f = & \bar{\Psi}_Q (i\not{\partial} - M_Q) \Psi_Q + \bar{\Psi}_S (i\not{\partial} - M_S) \Psi_S \\ & + \frac{\lambda_R f}{\sqrt{2}} \bar{\Psi}_{tR} P_L (\epsilon_{tS} U \Psi_S + \epsilon_{tQ} U \Psi_Q) \\ & + \lambda_L f \bar{\Psi}_{qL} P_R (\epsilon_{qS} U \Psi_S + \epsilon_{qQ} U \Psi_Q) + h.c, \end{aligned}$$

The use of V

Rewrite the top partners into a full rep of G

$$\Psi_+ = V\Psi_- \quad V = \begin{pmatrix} \mathbf{1}_{4 \times 4} & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Psi_+ = \frac{1}{\sqrt{2}}(\Psi_2 + \Psi_1) \quad \Psi_- = \frac{1}{\sqrt{2}}(\Psi_2 - \Psi_1)$$

$$c_{\pm R} = \frac{\epsilon_{tQ} \pm \epsilon_{tS}}{2}, \quad c_{\pm L} = \frac{\epsilon_{qQ} \pm \epsilon_{qS}}{\sqrt{2}}.$$

The Lagrangian is G invariant except for V

Elementary-composite
mixing is G invariant

$$c_{-L} = c_{-R} = 0.$$

$$\begin{aligned} \mathcal{L}_f = & \bar{\Psi}_+ i \not{\partial} \Psi_+ + \bar{\Psi}_- i \not{\partial} \Psi_- + \lambda_R f c_{+R} \bar{\Psi}_{tR} P_L U \Psi_+ \\ & + \lambda_L f c_{+L} \bar{\Psi}_{qL} P_R U \Psi_+ - (M_Q + M_S) \bar{\Psi}_{+L} \Psi_{+R} \\ & - (M_Q - M_S) \bar{\Psi}_{+L} V \Psi_{+R} + h.c. \end{aligned} \quad (22)$$

Enlarged global sym:

$$SO(5)_L \times SO(5)_R$$

Symmetries in CS

Two vector mass: twisted and untwisted:

$$SO(5)_L \quad \Psi_{+L} \rightarrow g'_L \Psi_{+L}$$

$$SO(5)_R \quad \Psi_{+L} \rightarrow g'_L \Psi_{+L}$$

The mass term **explicitly** break the global symmetry

Maximal Symmetry: Only the Twisted Mass

$$M_Q - M_S = 0 \Rightarrow SO(5)_L \times SO(5)_R / SO(5)_V$$

$$M_Q + M_S = 0 \Rightarrow SO(5)_L \times SO(5)_R / SO(5)_{V'} \quad g'_L V g'^{\dagger}_R = V$$

$$|M_Q| \neq |M_S| \Rightarrow SO(5)_L \times SO(5)_R / SO(4)_V \quad (23)$$

The form factors

Integrating out the top partners, we have the form factors in the EFT

$$\begin{aligned}
 \frac{\Pi_0^{q,t}}{\lambda_{L,R}^2 f^2} &= 1 + \frac{(c_{-L,R}^2 + c_{+L,R}^2)(M_Q^2 + M_S^2 - 2p^2)}{2(p^2 - M_S^2)(M_Q^2 - p^2)} \\
 &+ \frac{c_{-L,R}c_{+L,R}(M_S + M_Q)(M_S - M_Q)}{(p^2 - M_S^2)(M_Q^2 - p^2)} \\
 \frac{\Pi_1^{q,t}}{\lambda_{L,R}^2 f^2} &= \frac{c_{+L,R}c_{-L,R}(M_Q^2 + M_S^2 - 2p^2)}{(p^2 - M_S^2)(M_Q^2 - p^2)} \\
 &+ \frac{(c_{+L,R}^2 + c_{-L,R}^2)(M_S - M_Q)(M_S + M_Q)}{2(p^2 - M_S^2)(M_Q^2 - p^2)} \\
 \frac{M_1^t}{\lambda_L \lambda_R f^2} &= \frac{M_Q^2 M_S (c_{-L} - c_{+L})(c_{-R} - c_{+R})}{2(p^2 - M_Q^2)(p^2 - M_S^2)} \\
 &- \frac{M_S^2 M_Q (c_{-L} + c_{+L})(c_{-R} + c_{+R})}{2(p^2 - M_Q^2)(p^2 - M_S^2)} \\
 &+ \frac{M_Q (c_{-L} + c_{+L})(c_{-R} + c_{+R}) p^2}{2(p^2 - M_Q^2)(p^2 - M_S^2)} \\
 &- \frac{M_S (c_{-L} - c_{+L})(c_{-R} - c_{+R}) p^2}{2(p^2 - M_Q^2)(p^2 - M_S^2)}, \tag{54}
 \end{aligned}$$

Vh structure from symmetry

$SO(5)_V$

UV finite

c_{+L} is turned off, Higgs shift symmetry $h^{\hat{a}} \rightarrow h^{\hat{a}} + \alpha^{\hat{a}}$

$$\Psi_{+L} \rightarrow g'_L \Psi_{+L}$$

subgroup of
transformation on the left

$$\Psi_{+R} \rightarrow V g'_L \Psi_{+R}$$

$$(g'_L = \exp(i\alpha^{\hat{a}} T^{\hat{a}}))$$

$$|\lambda_L \lambda_R|^2 c_{+L}^2 c_{+R}^2 f^4 (M_1 - M_2)^2 / \Lambda^2. \quad (26)$$

Top mass square!

$$m_t = c_{+L} c_{+R} (M_Q - M_S) f^2 / (2M_T M_{T_1})$$

$SO(4)_V$

Log divergent

$$V_{L\xi} \sim |\lambda_L|^2 c_{+L}^2 f^2 (M_1 + M_2)(M_1 - M_2) \log \Lambda^2 \quad (24)$$

Higgs potential tuning

$$V_f(h) \simeq -\gamma_f s_h^2 + \beta_f s_h^4,$$

$$\xi = \frac{\gamma_f}{2\beta_f}$$

To obtain $\xi \ll 1$

γ_{div} much smaller than β_{div}

$$\begin{aligned} V_{\text{div}} &= \frac{N_c M_f^4}{16\pi^2 g_f^2} \left[\left(\frac{c_L}{2} \epsilon_L^2 - c_R \epsilon_R^2 + c_{LL} \frac{\epsilon_L^4}{g_f^2} + c_{RR} \frac{\epsilon_R^4}{g_f^2} \right) s_h^2 \right. \\ &\quad \left. + \left(c'_{LL} \frac{\epsilon_L^4}{g_f^2} + c'_{RR} \frac{\epsilon_R^4}{g_f^2} \right) s_h^4 \right] \\ &\equiv -\gamma_{\text{div}} s_h^2 + \beta_{\text{div}} s_h^4 \end{aligned} \quad (34)$$

Large finite piece from Π_1 form factor (expansion over s_h or c_h) tends to make $\gamma \gg \beta$

log divergent

$\mathcal{O}(\epsilon_L^4)$ and $\mathcal{O}(\epsilon_R^4)$.

quadratic divergent parts

$\mathcal{O}(\epsilon_L^2)$ and $\mathcal{O}(\epsilon_R^2)$

$c_L \sim c_R \sim \Lambda^2$

$c_{LL} \sim c'_{LL} \sim c_{RR} \sim c'_{RR} \sim \log \Lambda$

If UV divs cancels but finite remains

$$\Delta^{5+5} \simeq \frac{1}{\xi} \frac{g_f^2}{\epsilon^2}$$

Double tuning

Tunnings in EWSB

$$\begin{aligned}
 V_h &= c_{LR} \frac{N_c M_f^4}{16\pi^2} \left(\frac{\epsilon_L^2 \epsilon_R^2}{g_f^4} \right) [-s_h^2 + s_h^4] + \mathcal{O}\left(\frac{\epsilon_L^4 \epsilon_R^4}{g_f^8}\right) \\
 &\simeq c_{LR} \frac{N_c M_f^4}{16\pi^2} \left(\frac{y_t}{g_f} \right)^2 [-s_h^2 + s_h^4] + \mathcal{O}\left(\frac{y_t^4}{g_f^4}\right) \\
 &\equiv -\gamma_f s_h^2 + \beta_f s_h^4 \quad (39)
 \end{aligned}$$

Maximally
symmetric case

$$\xi = \frac{\gamma_f}{2\beta_f} = 0.5$$

Cancellation from the gauge sector

$$\gamma_g = -\frac{9f^2 g^2 m_\rho^2 \log 2}{64\pi^2}$$

$\xi \ll 1$, we require $\gamma_f \simeq -\gamma_g$.

Assuming 1st & 2nd Weinberg sum rule, UV finite

$$\Delta^{(5+5)} = \frac{\max(|\gamma_f|, |\gamma_g|)}{|\gamma_f + \gamma_g|} \simeq \max\left(\frac{1}{2\xi}, \frac{1}{2\xi} - 1\right) = \frac{1}{2\xi} \quad (44)$$

20% tuning

How to get 125 GeV Higgs?

$$m_t \sim \sin \theta_L \sin \theta_R |M_Q - M_S| s_h$$

Usually top is too heavy, difficult to get a light Higgs

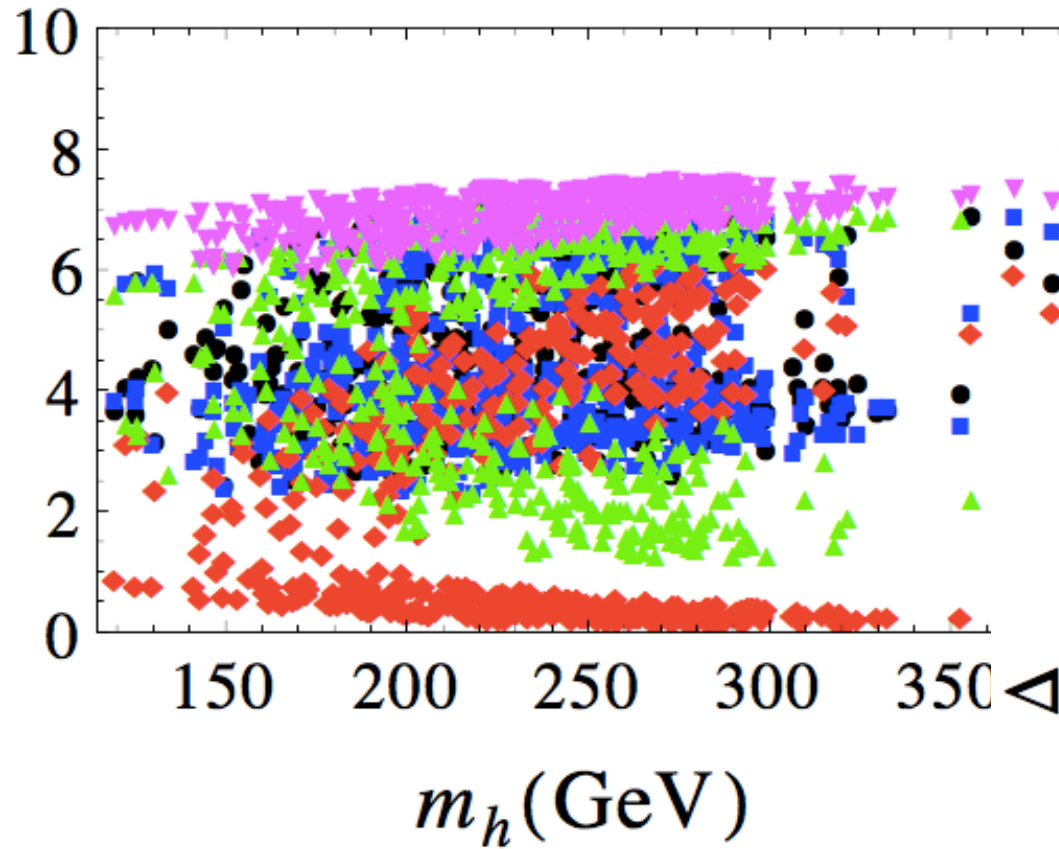
$$\theta_L \text{ and } \theta_R \quad \text{minimal}$$

$$M_Q = -M_S$$

$$\min\{M_T, M_{T_1}\} = \min\left\{\frac{M_S}{\cos \theta_L}, \frac{M_Q}{\cos \theta_R}\right\} \quad \text{minimal}$$

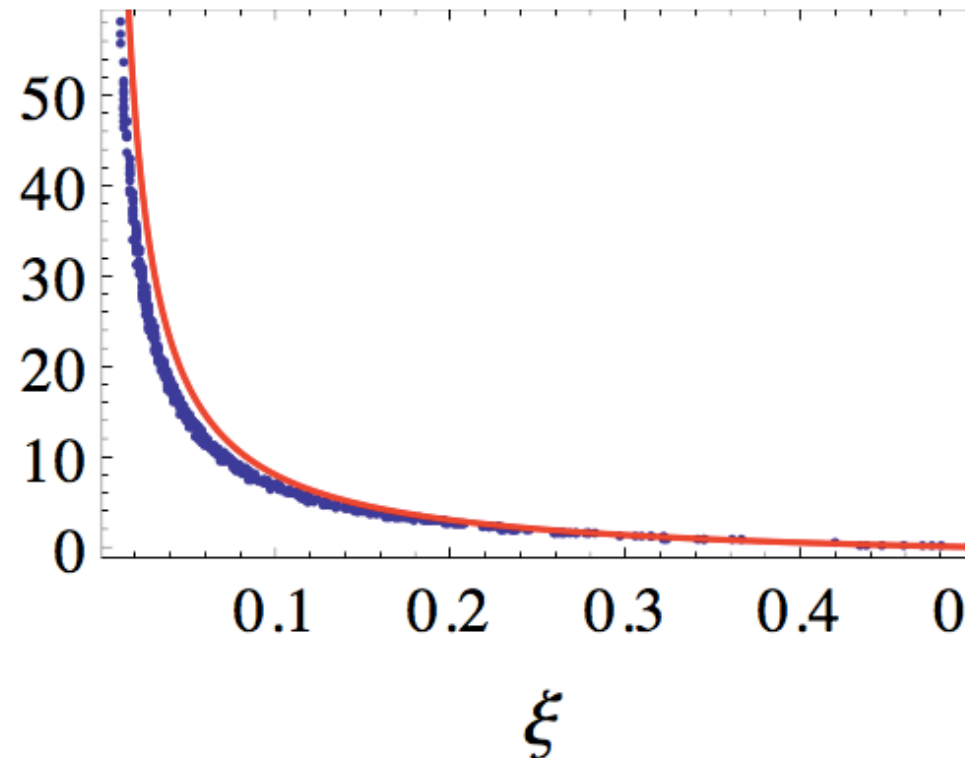
$$m_H \propto \min\{M_T, M_{T_1}\} m_t / f$$

Numerical tuning



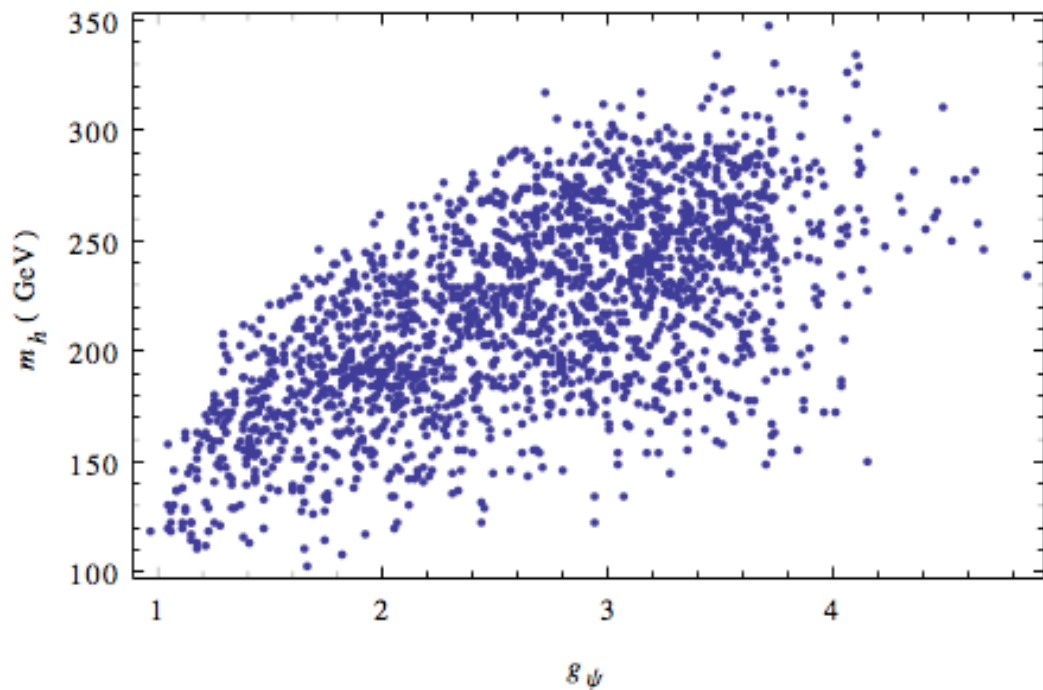
$$\Delta_m \simeq 2\gamma_g/|\gamma_f + \gamma_g| \simeq \frac{1}{\xi} - 2$$

One free parameter except f

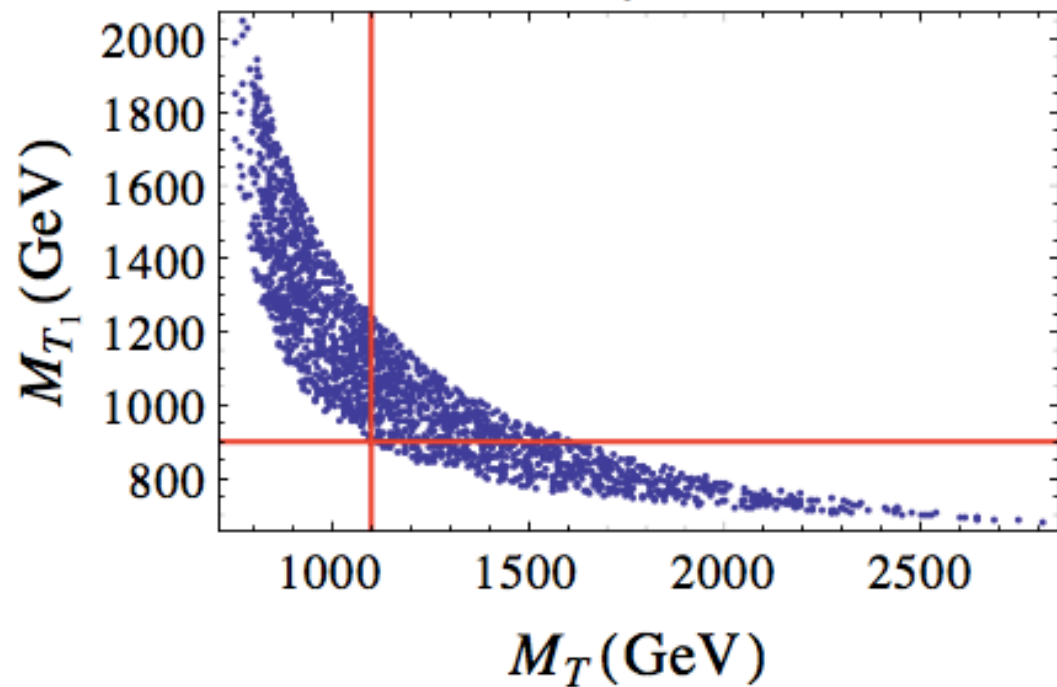


$$\Delta = \max \left| \frac{2x_i}{s_h} \frac{c_h^2}{f^2 m_h^2} \frac{\partial^2 V}{\partial x_i \partial s_h} \right|$$

Scan



One free parameter
except for f

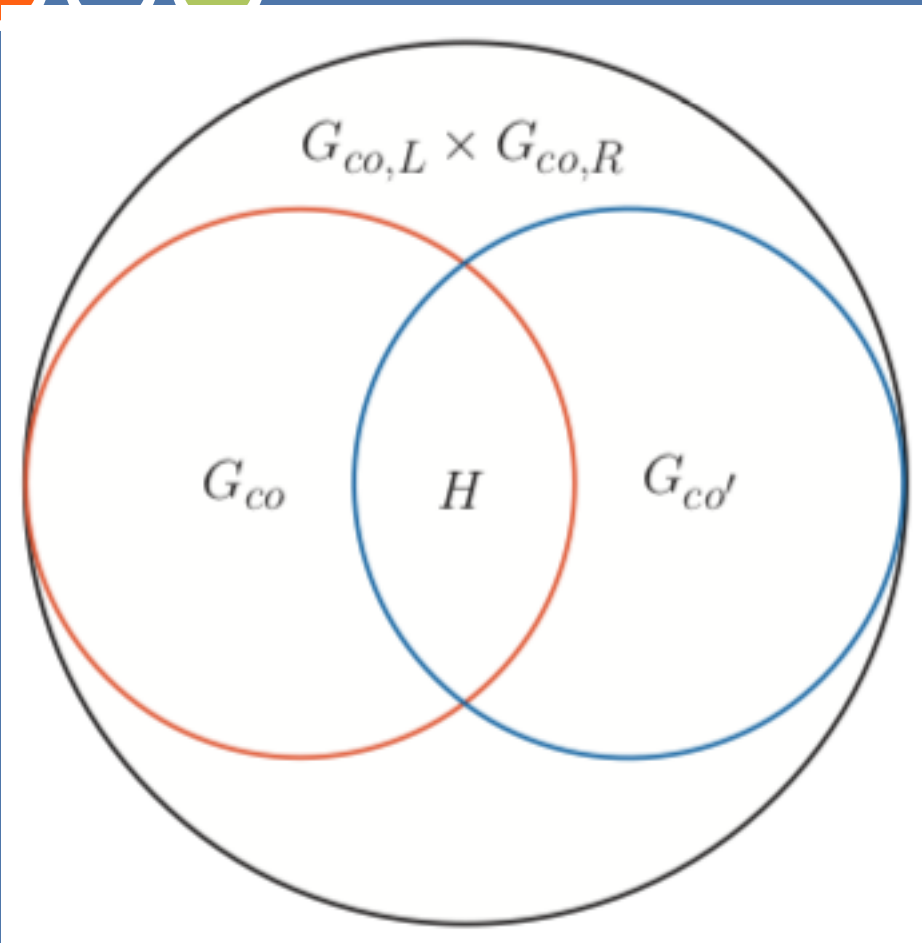


A decorative graphic on a blue background. It features a large white speech bubble in the center containing the title. To the left of the bubble is a large orange circle, a smaller white circle, and a green circle. To the right is a green circle and a large blue circle. All circles are connected to the central bubble by white lines.

Emergence of Maximal Symmetry

C. Csaki, T. Ma, [J. Shu](#), J-H. Yu, arxiv:1810.07704

Why a maximal symmetry?



sounds like symmetry inflow

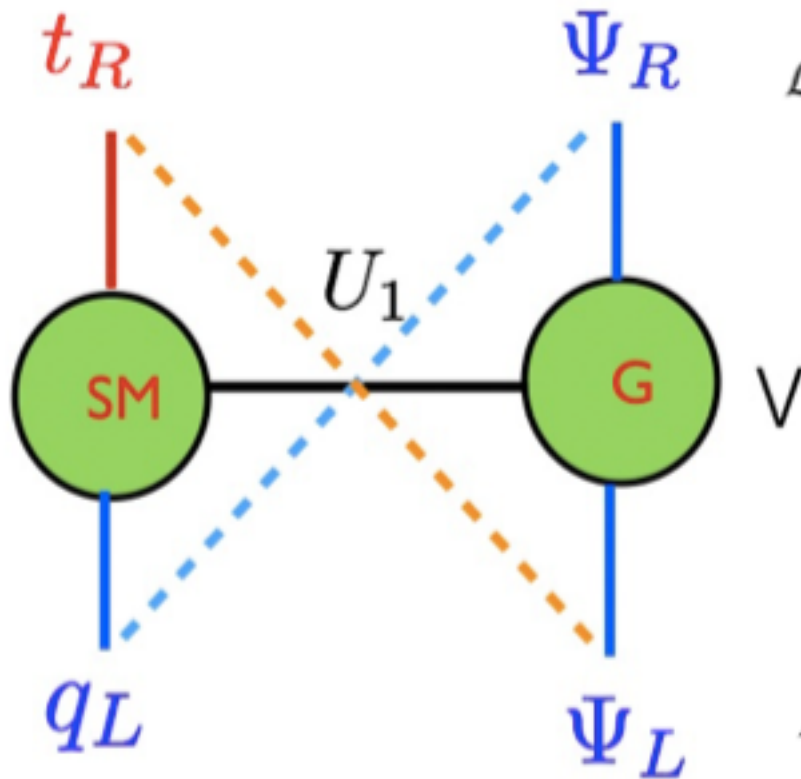
Ordinary case **global** symmetry breaks into H at the boundary

Maximal symmetry: **global** symmetry breaks into $G_{\{V\}}$ at the boundary

Integrating out the bulk from this boundary (composite) **preserve this global symmetry** and **transmitted** it to the other boundary (SM elementary)

How to realize a maximal symmetry?

Bulk fermion:



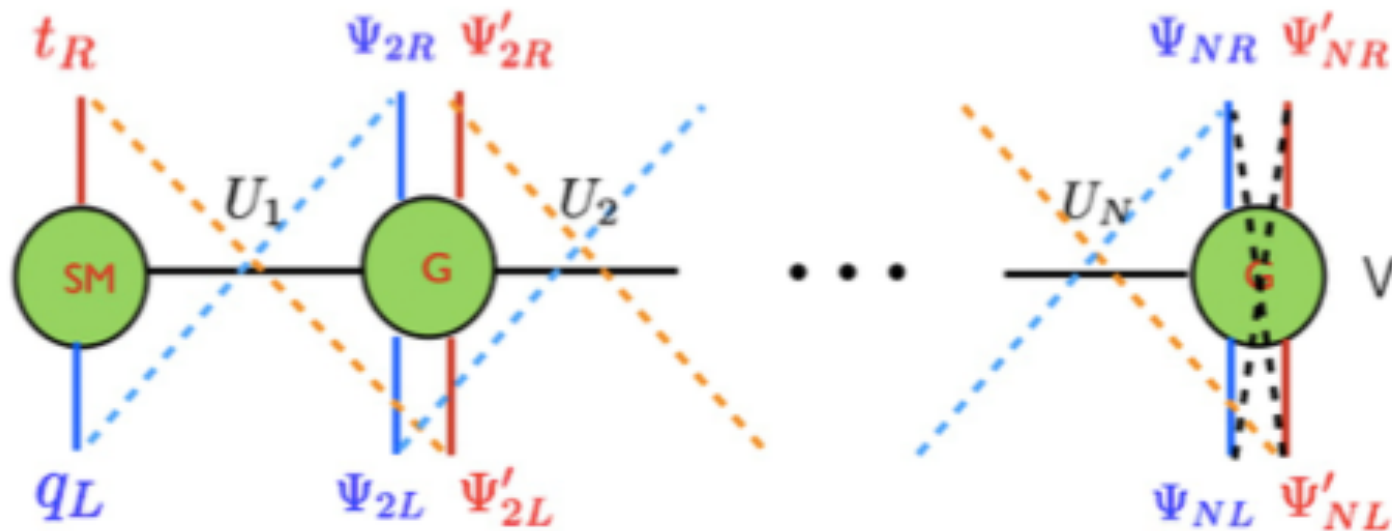
$$\mathcal{L}_{\text{eff}} = \bar{\Psi}_{q_L} \not{p} (\Pi_0^L(p) + \Pi_1^L(p) \Sigma') \Psi_{q_L} - \bar{\Psi}_{q_L} M_1^t(p) \Sigma' \Psi_{t_R} + \bar{\Psi}_{t_R} \not{p} (\Pi_0^R(p) + \Pi_1^R(p) \Sigma') \Psi_{t_R} + h.c. , \quad (2)$$

Both LH & RH fields are in the **fundamental** representation

UV completion of two site moose

$$\mathcal{L}_f = \bar{q}_L i \not{D} q_L + \bar{\Psi} i \not{D} \Psi + \bar{t}_R i \not{D} t_R - \epsilon_L \bar{\Psi}_{q_L} U_1 \Psi_R - M \bar{\Psi}_L \Sigma \Psi_R - \epsilon_R \bar{\Psi}_L U_1^\dagger \Psi_{t_R} + h.c. \quad (5)$$

How to realize a maximal symmetry?

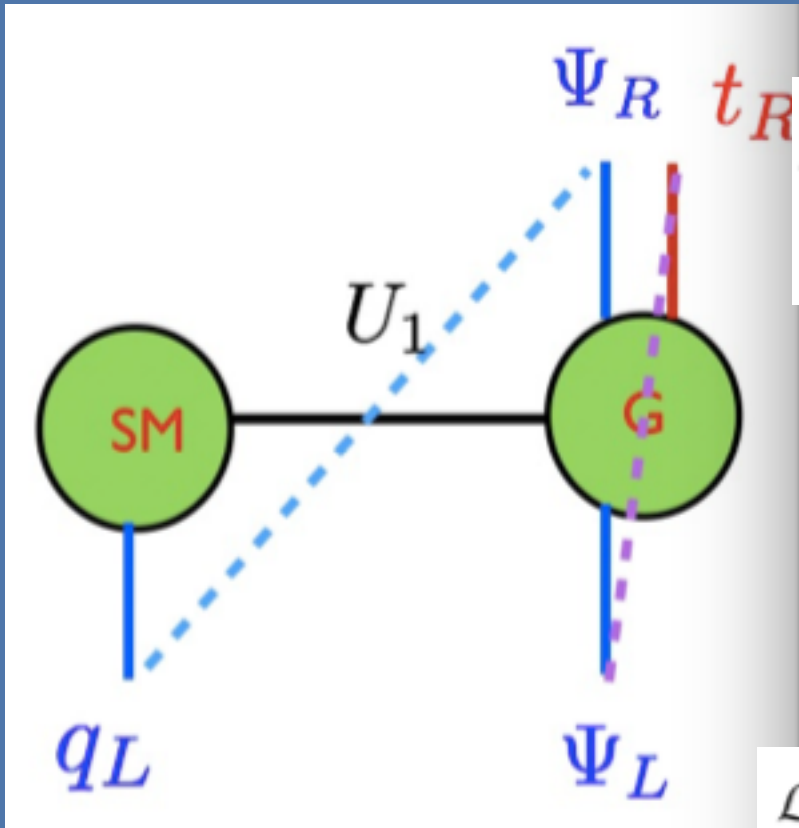


$$\mathcal{L}_f = \bar{q}_L i \not{D} q_L + \bar{t}_R i \not{D} t_R + \sum_{i=2}^N (\bar{\Psi}_i i \not{D} \Psi_i)$$

$$\begin{aligned}
 & - \epsilon_1 \bar{\Psi}_{qL} U_1 \Psi_{2R} - \sum_{j=2}^{N-1} \epsilon_j \bar{\Psi}_{jL} U_j \Psi_{j+1R} - M_i \sum_{i=2}^N \bar{\Psi}_{iL} \Psi_{iR} \\
 & - \epsilon'_1 \bar{\Psi}_{tR} U_1 \Psi'_{2L} - \sum_{j=2}^{N-1} \epsilon'_j \bar{\Psi}'_{jL} U_j \Psi'_{j+1R} - M'_i \sum_{i=2}^N \bar{\Psi}'_{iL} \Psi'_{iR} \\
 & - M (\bar{\Psi}_{NL} \Sigma \Psi'_{NR} + \bar{\Psi}'_{NL} \Sigma \Psi_{NR}) + h.c. \tag{11}
 \end{aligned}$$

Why a maximal symmetry?

Boundary fermion:



$$\mathcal{L}_{\text{eff}} = \bar{\Psi}_{q_L} \not{p} (\Pi_0^L(p) + \Pi_1^L(p) \Sigma') \Psi_{q_L} + \bar{\Psi}_{t_R} \not{p} \Pi_0^R(p) \Psi_{t_R} + \bar{\Psi}_{q_L} M_1^t(p) U \Psi_{t_R} + h.c. \quad (3)$$

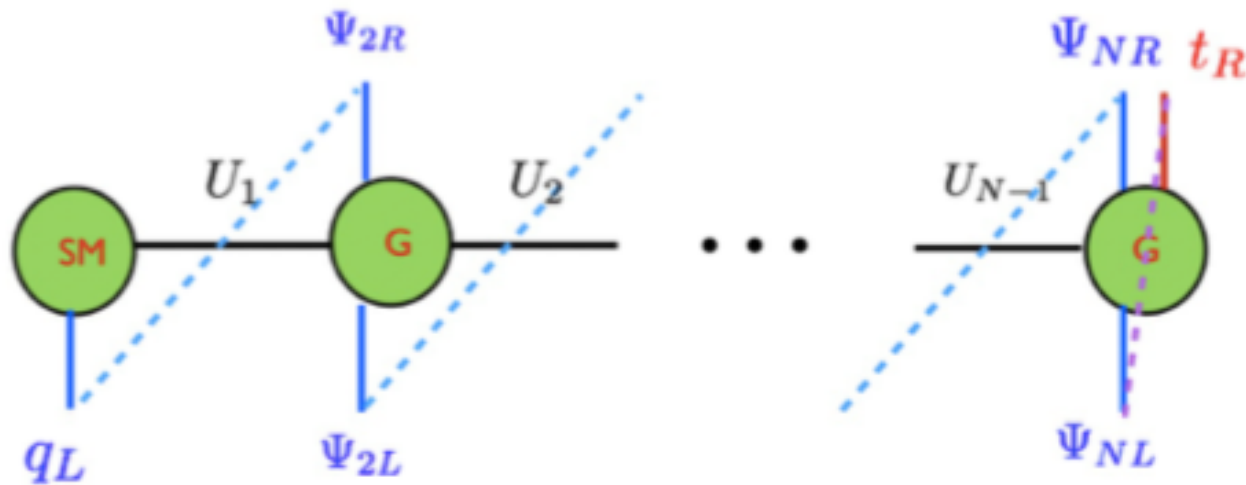
RH SM fermions are **singlets**

UV completion of two site moose

$$\mathcal{H} = U^\dagger \mathcal{V} \text{ with } \mathcal{V} = (0, 0, 0, 0, 1).$$

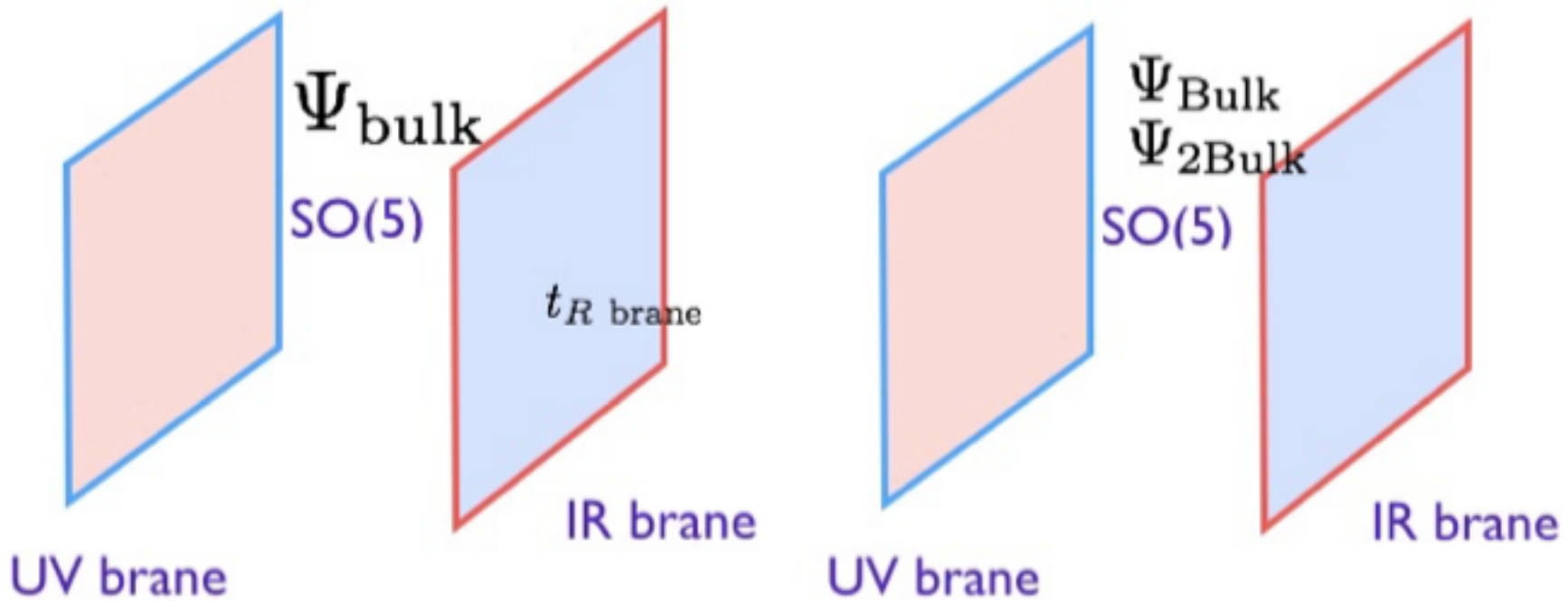
$$\mathcal{L}_f = \bar{q}_L i \not{D} q_L + \bar{\Psi} i \not{D} \Psi + \bar{t}_R i \not{D} t_R - \epsilon_L \bar{\Psi}_{q_L} U_1 \Psi_R - M \bar{\Psi}_L \Psi_R - \epsilon_R \bar{\Psi}_L \mathcal{H}' t_R + h.c. \quad (6)$$

Why a maximal symmetry?



$$\begin{aligned}
 \mathcal{L}_f &= \bar{q}_L i \not{D} q_L + \sum_{i=2}^N \bar{\Psi}_i i \not{D}_i \Psi_i + \bar{t}_R i \not{D} t_R \\
 &- \epsilon_1 \bar{\Psi}_{q_L} U_1 \Psi_{2R} - \sum_{j=2}^{N-1} \epsilon_j \bar{\Psi}_{jL} U_j \Psi_{j+1R} \\
 &- \sum_{i=2}^N M_i \bar{\Psi}_{iL} \Psi_{iR} - \epsilon_N \bar{\Psi}_{NL} \mathcal{H}' t_R + h.c. \quad (7)
 \end{aligned}$$

Extra dimension case:



$$U(R, R') = \text{Exp}\left(i \frac{-\sqrt{2}\pi \hat{a} T^{\hat{a}}}{f}\right),$$

After integrating out the bulk

$$\mathcal{L}_{\text{eff}} = \bar{\chi}'_L \not{p} \Pi_L(p) \chi'_L + \bar{t}_R \not{p} t_R + (M(p) \bar{\chi}'_L \mathcal{H} t_R + h.c.)$$

$$\mathcal{L}_H = \bar{\chi}_L \not{p} \Pi^L(\tilde{m}) \chi_L - \bar{\psi}_R \not{p} \Pi^R(\tilde{m}) \psi_R + M^{LR} (\bar{\chi}_L V \psi_R + \bar{\psi}_R V \chi_L),$$

Comment

What I feel interesting or critical is that:

The **boundary symmetry** completely controls the **bulk pNGB properties**, in particular, the UV sensitivity of the pNGB Coleman-Weinberg potential

I wonder if there is a application in condense matter physics?

MS in the lattice can also be applied to low-dim condense matter system (Bilayer Quantum Hall System?)

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Naturalness Sum rules

C. Csaki, F. Freitas, L. Huang, T. Ma, [J. Shu](#), M. Perelstein, arxiv:1811.01961

Test and predictions

Top kinetic terms: no corrections from Higgs

$$M_t(h) \sim \sin\left(\frac{2h}{f}\right) \left(1 + \frac{1}{2} \sin^2(h/f) (\Pi_1^q(0) - \Pi_1^t(0))\right)$$

C. Csaki, T. Ma, J. Shu., 1702.00405
D. Liu, I. Low, C. Wagner, 1703.07791

However, the ggh coupling only scale with the derivative of the first part.

$$c_g = c_t$$

Maximal Symmetry limit

100 TeV perhaps tth 1%

M. Mangano, T Plehn, P. Reimitz, T. Schell, H-S. Shao, 1507.08169

Test and predictions

Find the top partner resonance (charge 2/3), sum rule of diagonal Higgs Yukawa & mass

Mass eigenstates

$$\text{Tr}[Y_m M_D] = 0 + \mathcal{O}(v^2)$$

C. Csaki, T. Ma, Phys.Rev.Lett. 119 (2017) no13, 131803

C-R Chen, J. Hajer, T. Liu, I. Low, H. Zhang,
JHEP 1709 (2017) 129

No quadratic div

$$\text{Tr}[Y_m M_D^3] = 0 + \mathcal{O}(v^2/M_f^2)$$

No log div

C. Csaki, T. Ma, J. Shu., Phys.Rev.Lett. 119 (2017) no13, 131803

$$M_Q + M_S = 0$$

Lightest exotic charge (5/3)

Gauge Sum rules

Quadratic divergence

$$\text{Tr}[g_{VVh}] = 0 + \mathcal{O}(\tilde{v}^2/f^2).$$

Log divergence

$$\text{Tr}[g_{VVh}M_V^2] = 0 + \mathcal{O}(\tilde{v}^2/f^2),$$

SUSY Case

$$\text{Tr}[g_{SSH}] - 2\text{Tr}[Y_M M_D^\dagger + M_D Y_M^\dagger] + 3\text{Tr}[g_{VVh}] = 0,$$

$$\text{Tr}[g_{SSH}] - 4\text{Tr}[Y_M M_D] + 3\text{Tr}[g_{VVh}] = 0.$$

Quadratic divergence

Top sector/stop sector

$$\sum_i g_{\tilde{t}_i \tilde{t}_i h} - 4y_t m_t = 0,$$

Gauge/gaugino/Higgs/Higgsino sector

$$4 \sum_i (y_{C_i^+ C_i^- h} m_{C_i} + y_{N_i N_i h} m_{N_i}) - 3(g_{W^+ W^- h} + g_{ZZh}) \\ - \sum_i (g_{H_i^0 H_i^0 h} + g_{H_i^+ H_i^- h}) - g_{hhh} = 0$$

Non-SUSY Case: Collider



Non-susy case done with signs! See the talk tomorrow!

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The build-in twin Higgs (Trigonometric Parity)

C. Csaki, T. Ma, [J. Shu.](#), Phys.Rev.Lett. 121 (2018) no23, 231801

Why twin Higgs?

Z. Chacko, H.-S. Goh, R. Harnik, Phys.Rev.Lett. 96 (2006) 231802

The key reason is that we still do not see the colored top partner yet!

Colored top partners are the **most sensitive probe** of composite Higgs models

Proved upper limit of lightest top partners for given symmetry breaking scale f

Light Higgs  **Light Top Partners**

D. Marzocca, M. Serone, J. Shu., JHEP 1208, 013 (2012)
O.Matsedonskyi, G. Panico, A. Wulzer, JHEP 1301, 164 (2013)

EW charged twin top almost have **zero LHC bounds**

See for instance

Neutral Naturalness

N. Craig, A.Katz, M.Strassler, R. Sundrum, JHEP 1507, 105 (2015)

Why composite twin Higgs?

Funny trigonometric parity $s_h \leftrightarrow c_h$.

References added soon

- Why Twin Higgs? **Highly constrained by LEP**
- The radiatively generated Higgs potential
- universal prediction on Higgs couplings (Like pion soft theorem)

If the **strong dynamics** triggers the breaking G/H ,
pNGB is a **composite** particle.

QCD chiral symmetry, pion

Trigonometric Parity as the build-in Twin Parity

However, the goldstone itself does have the spontaneous broken symmetry!

The symmetry of the G/H coset space manifold!

Inside any coset space manifold, there is a trigonometric parity

Physical higgs has a shift symmetry in the corresponding unbroken direction

C. Csaki, T. Ma, J. Shu., Phys.Rev.Lett. 121 (2018) no23, 231801

$$\pi^i / f \rightarrow \pi^i / f + \epsilon^i.$$

$$\frac{\pi^i}{f} \rightarrow -\frac{\pi^i}{f} + \frac{\pi}{2}$$

Higgs parity:

$$\pi^i \rightarrow -\pi^i.$$

$$U(1) \sim SO(2)$$

$$SO(N+1)/SO(N) \quad S^N$$

$$U = \begin{pmatrix} \mathbb{1}_3 & & & \\ & \cos \frac{h}{f} & & \sin \frac{h}{f} \\ & & 1 & \\ & -\sin \frac{h}{f} & & \cos \frac{h}{f} \end{pmatrix}.$$

Exchange of the 4th and 6th row

Adding matter fields

The matter fields have to conserve such a build-in trigonometric parity

$$\Psi_{QL} = \frac{1}{\sqrt{2}} \begin{pmatrix} b_L \\ -ib_L \\ t_L \\ it_L \\ 0 \\ 0 \end{pmatrix}.$$

$$\Psi_{\tilde{t}_L} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \tilde{t}_L \\ i\tilde{t}_L \end{pmatrix},$$

Exchange the coordinates in the 3rd and 5th, 4th and 6th row

$$P = P_0 P_1^h = \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & & 1 & & \\ & & & & & 1 \\ & & 1 & & & \\ & & & 1 & & \end{pmatrix}$$

$$P_1^h = \begin{pmatrix} \mathbb{1}_3 & & \\ & 1 & \\ & & 1 \end{pmatrix}.$$

$$y_t = \tilde{y}_t$$

$$\Psi_{QL} \leftrightarrow P\Psi_{\tilde{t}_L}, \quad t_R \leftrightarrow \tilde{t}_R, \quad \Sigma \rightarrow P\Sigma$$

Fermion Lag

The top and bottom sector

$$\begin{aligned}\mathcal{L}_{eff}^t &= \bar{b}_L \not{p} \Pi_0^q(p) b_L + \bar{t}_L \not{p} (\Pi_0^q(p) + \Pi_1^q(p) c_h^2) t_L \\ &+ \bar{t}_R \not{p} \Pi_0^t(p) t_R + \bar{\tilde{t}}_L \not{p} (\Pi_0^q(p) + \Pi_1^q(p) s_h^2) \tilde{t}_L \\ &+ \bar{\tilde{t}}_R \not{p} \Pi_0^t(p) \tilde{t}_R - \frac{iM_1^t(p)}{\sqrt{2}} (\bar{t}_L t_R s_h + \bar{\tilde{t}}_L \tilde{t}_R c_h) + h.c.\end{aligned}$$

A UV Completion

$$SO(6)/SO(5) \simeq SU(4)/Sp(4)$$

The latter has the fermion condensation

	$Sp(2N)$	$SU(2)_L$	$U(1)_Y$	$SU(3)_c$	$U(1)_\eta$
(ψ_1, ψ_2)	\square	\square	$\mathbf{0}$	$\mathbf{1}$	$\mathbf{1}$
ψ_3	\square	$\mathbf{1}$	$-\frac{1}{2}$	$\mathbf{1}$	$-\mathbf{1}$
ψ_4	\square	$\mathbf{1}$	$\frac{1}{2}$	$\mathbf{1}$	$-\mathbf{1}$

Gauge sector automatically satisfy the Weinberg sum rule
Lowest chiral breaking operators at UV: 4-fermions dim 6.

SU(4)/Sp(4) matter content

$$\Psi_{Q_L} = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{0} & Q \\ -Q^T & \mathbf{0} \end{pmatrix} \text{ and } \Psi_{\tilde{t}_L} = \frac{1}{\sqrt{2}} \begin{pmatrix} i\tilde{t}_L\sigma^2 & 0 \\ 0 & \mathbf{0} \end{pmatrix}$$

$$U = \begin{pmatrix} c'\mathbb{1}_2 & i\sigma^2 h s' \\ i\sigma^2 h s' & c'\mathbb{1}_2 \end{pmatrix}$$

$$\eta : i(\psi_1\psi_2 + \psi_3\psi_4 - \psi_1^c\psi_2^c - \psi_3^c\psi_4^c)$$

$$\begin{aligned} \mathcal{L}_{eff}^t &= \bar{b}_L \not{p} \Pi_0^q(p) b_L + \bar{t}_L \not{p} (\Pi_0^q(p) - 2\Pi_1^q(p) s_h^2) t_L \\ &+ \bar{t}_R \not{p} \Pi_0^t(p) t_R + \bar{\tilde{t}}_R \not{p} \Pi_0^t(p) \tilde{t}_R \\ &+ \bar{\tilde{t}}_L \not{p} (\Pi_0^q(p) - 2\Pi_1^q(p) c_h^2) \tilde{t}_L \\ &- \sqrt{2} M_1^t(p) \left(\bar{t}_L t_R s_h + \bar{\tilde{t}}_L \tilde{t}_R c_h \right) + h.c. \quad (36) \end{aligned}$$

Extension for Composite Top

	$Sp(2N)$	$SU(2)_L$	$U(1)_Y$	$SU(3)_c$	$SU(3)'_c$
χ_L	\square	1	$\frac{2}{3}$	\square	1
χ_R^c	\square	1	$-\frac{2}{3}$	$\bar{\square}$	1
$\tilde{\chi}_L$	\square	1	1	1	\square
$\tilde{\chi}_R^c$	\square	1	1	1	$\bar{\square}$

	$SU(4) \times SU(12)$	$Sp(4) \times SO(12)$
$\chi(\psi\psi)$	(6, 12)	(5, 12), (1, 12)
$\chi(\psi^c\psi^c)$	(6, 12)	(5, 12), (1, 12)
$\psi(\chi\psi)$	(10, 12)	(10, 12)
$\psi(\chi^c\psi^c)$	(1, $\bar{12}$)	(1, 12)
$\psi(\chi^c\psi^c)$	(15, $\bar{12}$)	(15, 12)

Extension for Composite Top

$$\mathcal{L} = f\bar{\Psi}_L U(\epsilon_{5L}\Psi_{5R} + \epsilon_{1L}\Psi_{1R}) + f\epsilon_R\bar{\Psi}_R\Psi_{1L} \\ + M_5\bar{\Psi}_{5L}\Psi_{5R} + M_1\bar{\Psi}_{1L}\Psi_{1R} + h.c.,$$

$$\Psi_Q = \begin{pmatrix} iB - iX_{5/3} \\ B + X_{5/3} \\ T + X_{2/3} \\ -T + X_{2/3} \\ iT'_+ - iT'_- \\ 0 \end{pmatrix} \quad \Psi_S = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ T'_+ + T'_- \end{pmatrix}$$

$$\tilde{\Psi}_Q = \begin{pmatrix} i\tilde{B}_{-1} - i\tilde{X}_1 \\ \tilde{B}_{-1} + \tilde{X}_1 \\ \tilde{T}_0 + \tilde{X}_0 \\ -\tilde{T}_0 + \tilde{X}_0 \\ i\tilde{T}'_+ - i\tilde{T}'_- \\ 0 \end{pmatrix} \quad \tilde{\Psi}_S = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \tilde{T}'_+ + \tilde{T}'_- \end{pmatrix}$$

Higgs potential

$$V_g = \gamma_g s_h^2 \quad V_f = \gamma_f (-s_h^2 + s_h^4),$$

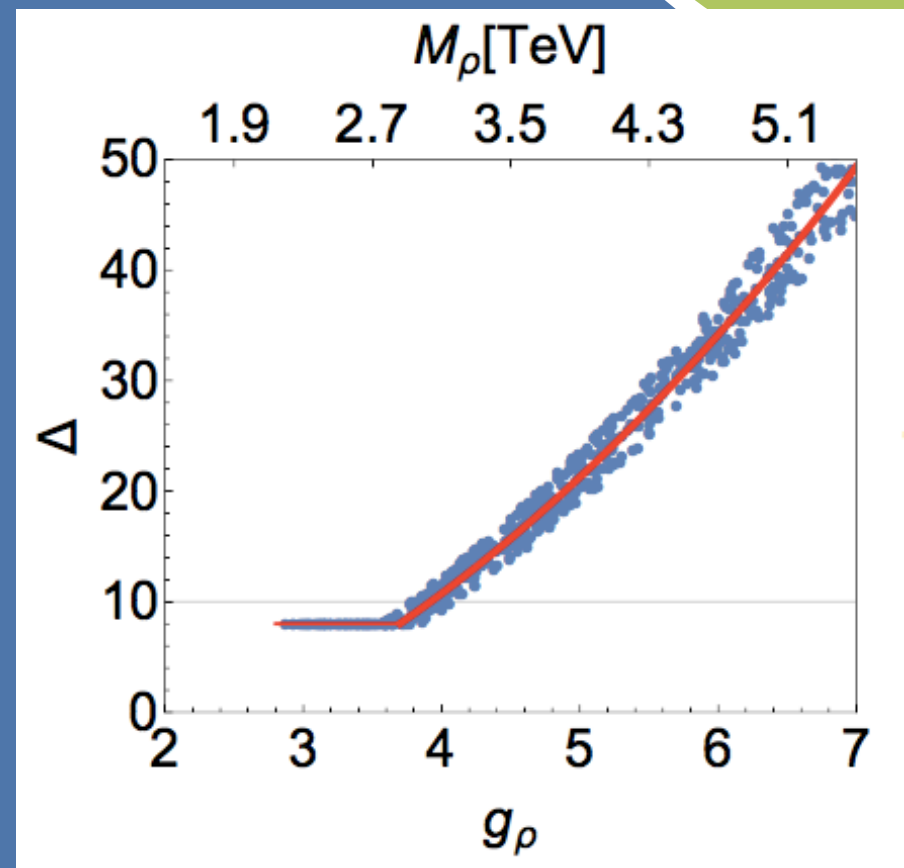
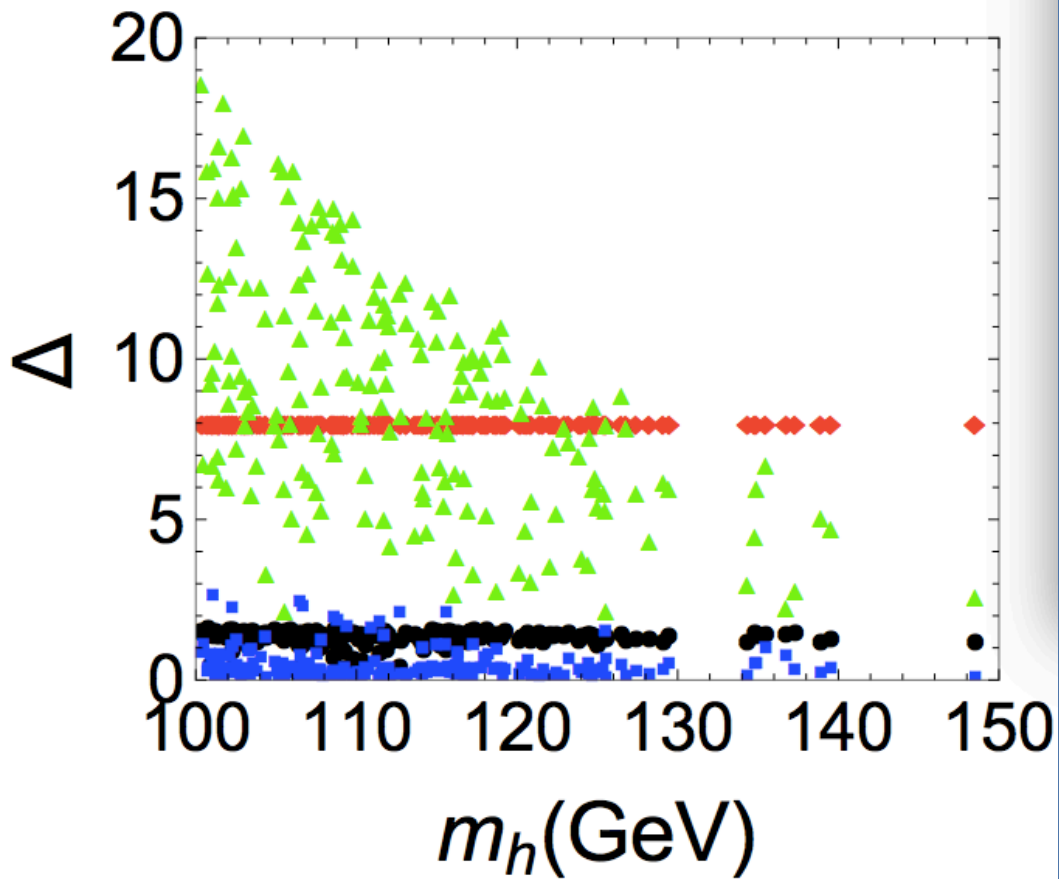
$$V = V_g + V_f = -\gamma s_h^2 + \beta s_h^4,$$

$$\gamma = \gamma_f - \gamma_g \text{ and } \beta = \gamma_f$$

$$\begin{aligned} V_f &\simeq c' \frac{N_c M_f^4}{16\pi^2} \left(\frac{y_t}{g_f}\right)^4 [-s_h^2 + s_h^4] \\ &\simeq c' \frac{N_c f^4}{16\pi^2} y_t^4 [-s_h^2 + s_h^4], \end{aligned}$$

Notice top
Yukawa 4th power

Higgs potential



Novel six top signals

$$t' \rightarrow B'_{\mu} t \rightarrow t \bar{t} t.$$

Completely new and novel channels

H-Y. Han, L. Huang, T. Ma, [J. Shu](#), T. Tait, Y.C. Wu, arxiv:1812.11286

Future Prospects

- Understanding models of EWSB (real progress after 2000)
- EFT approach to EWSB, connect collider physics with true natural of EWSB
- Theoretical Framework can be applied to many other aspects? (Inflation, axion, condensed matter?)

A decorative graphic on a blue background. It features a central white rounded rectangle containing the text "Backup slice". To the left of the rectangle is a large orange circle, and below it is a smaller green circle. To the right of the rectangle is a green circle above a larger blue circle. All circles are connected to the central white area by thin white lines.

**Backup
slice**

Discreet Parities

Hidden additional Z_2 forbids the tuning term: (like composite twin Higgs)

M. Geller, O. Telem, PRL 114, 191801 (2015)

R. Barbieri, D. Greco, R. Rattazzi, A. Wulzer, JHEP 1508, 161 (2015)

M. Low, A. Tesi, L.T. Wang, PRD 91, 095012 (2015)

$S_h \Leftrightarrow -C_h$ in the Higgs potential

Can be realized under the following transformation

$$\begin{aligned}\Psi_{+L} &\rightarrow P_1 \Psi_{+L}, \quad \Psi_{+R} \rightarrow V P_1 V \Psi_{+R}, \quad U \rightarrow V U V P_1 V, \\ \Psi_{qL} &\rightarrow V \Psi_{qL} = \Psi_{qL}, \quad \Psi_{tR} \rightarrow P_2 \Psi_{tR} = \Psi_{tR}\end{aligned}\quad (28)$$

$$P_1 = \text{diag}(1_{3 \times 3}, \sigma_1), \quad P_2 = \text{diag}(1_{3 \times 3}, -\sigma_3).$$

Vector bosons

SO(5)/SO(4)

Consider one vector meson and one axi-vector meson

$$\rho_\mu \equiv \mathbf{6}$$

$$a_\mu \equiv \mathbf{4}$$

$$\rho_\mu \rightarrow h\rho_\mu h^\dagger + \frac{i}{g_\rho} h\partial_\mu h^\dagger$$

$$a_\mu \rightarrow ha_\mu h^\dagger$$

The Lag based on HLS

$$\mathcal{L}^v = -\frac{1}{4} \text{Tr}[\rho_{\mu\nu}\rho^{\mu\nu}] + \frac{f_\rho^2}{2} \text{Tr}[(g_\rho\rho_\mu - E_\mu)^2]$$

$$\mathcal{L}^a = -\frac{1}{4} \text{Tr}[a_{\mu\nu}a^{\mu\nu}] + \frac{f_a^2}{2\Delta^2} \text{Tr}[(g_a a_\mu - \Delta d_\mu)^2]$$

$$\mathcal{L}_{\text{kin}} = \frac{f^2}{4} \text{Tr}[d_\mu d^\mu] \quad (3)$$

$$\rho_{\mu\nu} = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu - ig_\rho[\rho_\mu, \rho_\nu],$$
$$a_{\mu\nu} = \Delta_\mu a_\nu - \Delta_\nu a_\mu, \quad \Delta = \partial - iE.$$

$$m_\rho^2 = g_\rho^2 f_\rho^2 \quad m_a^2 = \frac{g_a^2 f_a^2}{\Delta^2}$$

Δ is a free parameter

Vector bosons

Further simplified as

$$\mathcal{L} = \frac{f^2 + 2f_a^2}{4} \text{Tr}[d_\mu d^\mu] - m_a f_a \text{Tr}[a_\mu (E_\mu + d_\mu)] + \frac{g_a^2 f_a^2}{2\Delta^2} \text{Tr}[a_\mu a^\mu] \\ + \frac{f_\rho^2}{2} \text{Tr}[E_\mu E^\mu] - m_\rho f_\rho \text{Tr}[\rho_\mu (E_\mu + d_\mu)] + \frac{g_\rho^2 f_\rho^2}{2} \text{Tr}[\rho_\mu \rho^\mu] \quad (7)$$

For symmetric coset space, G-invariant building blocks

$$\rho_\mu \pm a_\mu$$

$$E_\mu \pm d_\mu$$

$$\mathcal{L} = f_+^2 \text{Tr}[(d_\mu + E_\mu)^2] + f_-^2 \text{Tr}[V(E_\mu + d_\mu)V(E^\mu + d_\mu)] \\ - m_+^2 \text{Tr}[(\rho_\mu + a_\mu)(d_\mu + E_\mu)] - m_-^2 \text{Tr}[V(\rho_\mu + a_\mu)V(d_\mu + E_\mu)] \\ + \frac{m_\rho^2 + m_a^2}{4} \text{Tr}[(\rho_\mu + a_\mu)(\rho_\mu + a_\mu)] \\ + \frac{m_\rho^2 - m_a^2}{4} \text{Tr}[V(\rho_\mu + a_\mu)V(\rho_\mu + a_\mu)] \quad (8)$$

$$f_+^2 = \frac{f^2 + 2f_a^2 + 2f_\rho^2}{8},$$

$$m_+^2 = \frac{m_\rho f_\rho + m_a f_a}{2}, \quad m_-^2 = \frac{m_\rho f_\rho - m_a f_a}{2}$$

$$f_-^2 = \frac{f^2 + 2f_a^2 - 2f_\rho^2}{8}$$

Vector bosons



Again, theory is made of **one** G-invariant adjoints for **one** G and also V

$SO(5)_1$

$$U^\dagger D_\mu U \rightarrow \Omega_1 U^\dagger D_\mu U \Omega_1^\dagger$$

Higgs shift sym lies in $[SO(5)/SO(4)]_1$

$SO(5)_2$

$$\rho_\mu + a_\mu \rightarrow \Omega_2 (\rho_\mu + a_\mu) \Omega_2^\dagger$$

Automatically get the Weinberg sum rules

CYB in the 1st line

$$V_g \sim g_0^2 f_-^2 \Lambda^2$$

$$f_- = 0 \text{ 1st WS}$$

CYB in the 2nd line

$$V_g \sim g_0^2 m_+^2 m_-^2 \log \Lambda^2$$

$$m_- = 0 \text{ 2nd WS}$$

CYB in the 3rd line

$$V_g \sim g_0^2 m_+^4 (m_\rho^2 - m_a^2) / \Lambda^2$$

$$m_\rho \approx m_a$$

Higgs as pNGB

Consider the minimal group G/H

$$SO(5) \times U(1)_X \rightarrow SO(4) \times U(1)_X$$

K. Agashe, R. Contino, A. Pomarol, NPB 719 (2005) 165

at the scale $f > v$

$$\xi \equiv \frac{v^2}{f^2}$$

There are four NGBs: $\pi^{\hat{a}}$, with $\hat{a} = 1, 2, 3, 4$.

They transform as a **4 of $SO(4)$**

(2,2) of $SU(2) \times SU(2) \sim SO(4)$.

$$Y = T_{3R} + X$$

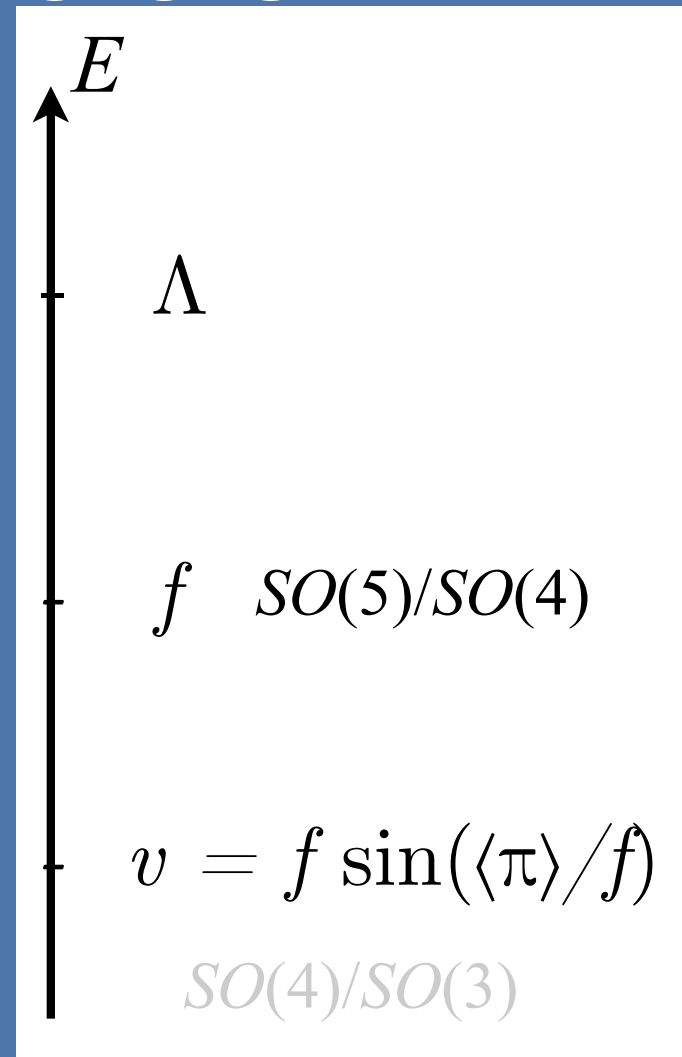
$$v = f \sin(\langle \pi \rangle / f)$$

$$SU(2)_L \times U(1)_Y \subset SU(2)_L \times SU(2)_R \times U(1)_X \sim SO(4)' \times U(1)_X$$

$SO(4)/SO(3)$

See other holographic models based on $SU(3)/SU(2)$

R. Contino, Y. Nomura, A. Pomarol, NPB 671 (2003) 148



CCWZ of GCHM

pNGB matrix:

$$U = \exp\left(i\frac{\sqrt{2}}{f}h^{\hat{a}}T^{\hat{a}}\right).$$

$$\hat{a} = 1, 2, 3, 4.$$

The CCWZ transformation

C.G. Callan, S.R. Coleman, J. Wess, B. Zumino, PR 177 (1969) 2247

$$U \rightarrow gU h(h^{\hat{a}}, g)^{\dagger}$$

$$iU^{\dagger}D_{\mu}U = \hat{d}_{\mu}^{\hat{a}}T^{\hat{a}} + \hat{E}_{\mu}T^a$$

SM gauge fields

Leading order chiral Lag

$$\mathcal{L}_{\sigma_g} = -\frac{1}{4}W_{\mu\nu}^{aL}W^{aL\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{f^2}{4}\text{Tr}\left(\hat{d}_{\mu}\hat{d}^{\mu}\right)$$

CCWZ of GCHM


$$iU^\dagger D_\mu U = \hat{d}_\mu^{\hat{a}} T^{\hat{a}} + \hat{E}_\mu T^a$$

SM gauged

$$\hat{d}_\mu = -\frac{\sqrt{2}}{f} (D_\mu h) + \dots$$

$$\hat{E}_\mu = g_0 A_\mu + \frac{i}{f^2} (h \overleftrightarrow{D}_\mu h) + \dots$$

Transform like a gauge field

$$m_W = \frac{gf}{2} \sin \frac{\langle h \rangle}{f} \equiv \frac{gv}{2}$$

$$s_h = \sin \frac{\langle h \rangle}{f}, \quad \xi \equiv s_h^2$$

Higgs physics

$$f^2 \sin^2 \frac{h}{f} = f^2 \left[\sin^2 \frac{\langle h \rangle}{f} + 2 \sin \frac{\langle h \rangle}{f} \cos \frac{\langle h \rangle}{f} \left(\frac{h}{f} \right) + \left(1 - 2 \sin^2 \frac{\langle h \rangle}{f} \right) \left(\frac{h}{f} \right)^2 + \dots \right]$$

$$= v^2 + 2v\sqrt{1-\xi} h + (1-2\xi) h^2 + \dots$$

W boson mass

modification of hVV
coupling

Similarly for fermions.

$$a = \sqrt{1-\xi}$$

$$b = 1 - 2\xi$$

$$m_f(h) \propto \sin \left(\frac{2h}{f} \right)$$

$$c = \frac{1-2\xi}{\sqrt{1-\xi}}$$

5, 10

$$m_f(h) \propto \sin \left(\frac{h}{f} \right)$$

$$c = \sqrt{1-\xi}$$

Spinorial 4

电弱对称破缺机制

Higgs 势能:

辐射修正

$$V_f(h) \simeq -\gamma_f s_h^2 + \beta_f s_h^4,$$

$$\sin^2 \langle H \rangle / f = \xi \ll 1$$

$$m_H^2 = 8\xi(1 - \xi)\beta.$$

$$\gamma_g = -\frac{3}{8(4\pi)^2} \int_0^\infty dp_E^2 p_E^2 \left(\frac{3}{\Pi_0} + \frac{c_X^2}{\Pi_B} \right) \Pi_1,$$
$$\beta_g = -\frac{3}{64(4\pi)^2} \int_{\mu_g^2}^\infty dp_E^2 p_E^2 \left(\frac{2}{\Pi_0^2} + \left(\frac{1}{\Pi_0} + \frac{c_X^2}{\Pi_B} \right)^2 \right) \Pi_1^2.$$

规范波色子贡献

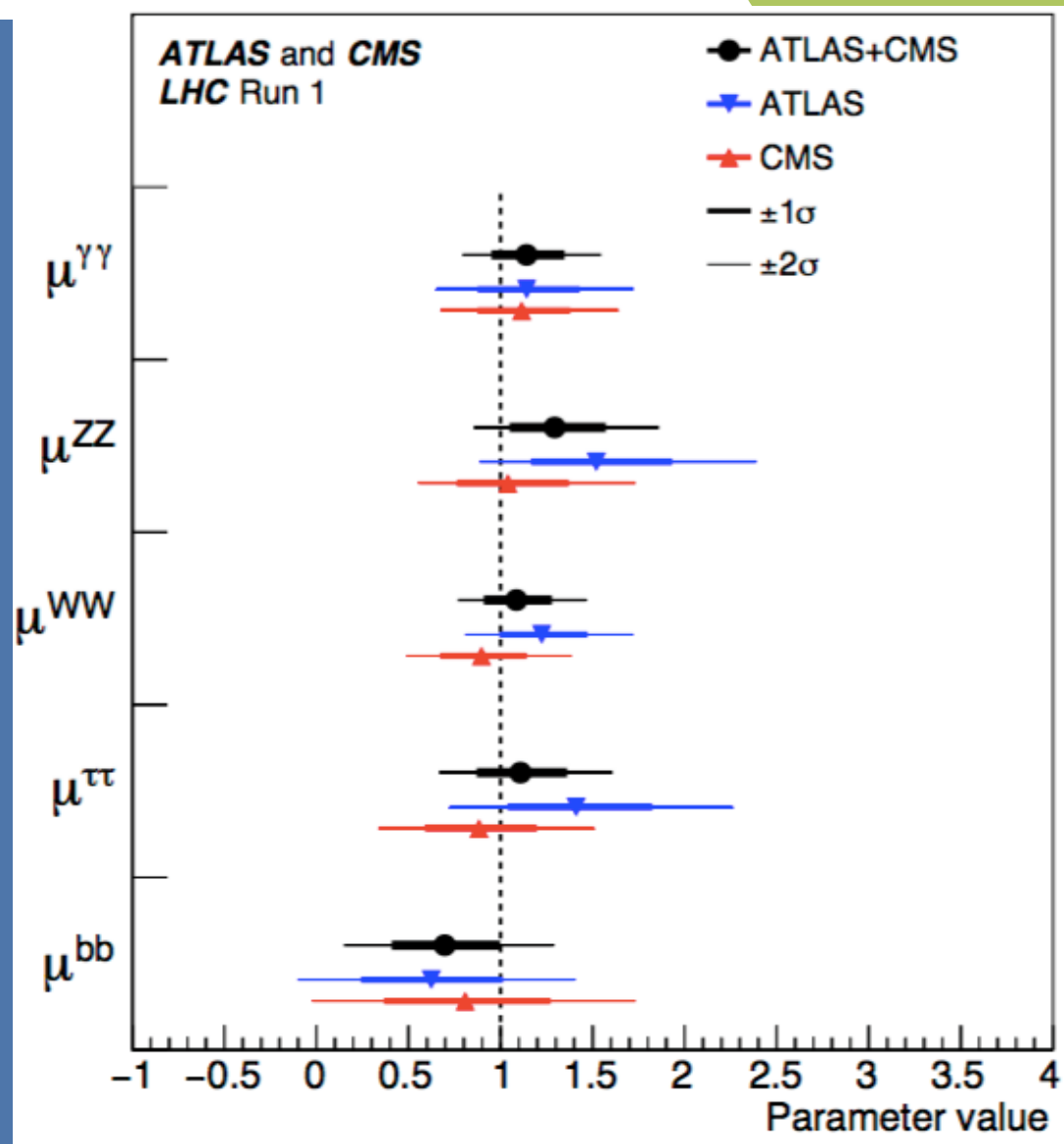
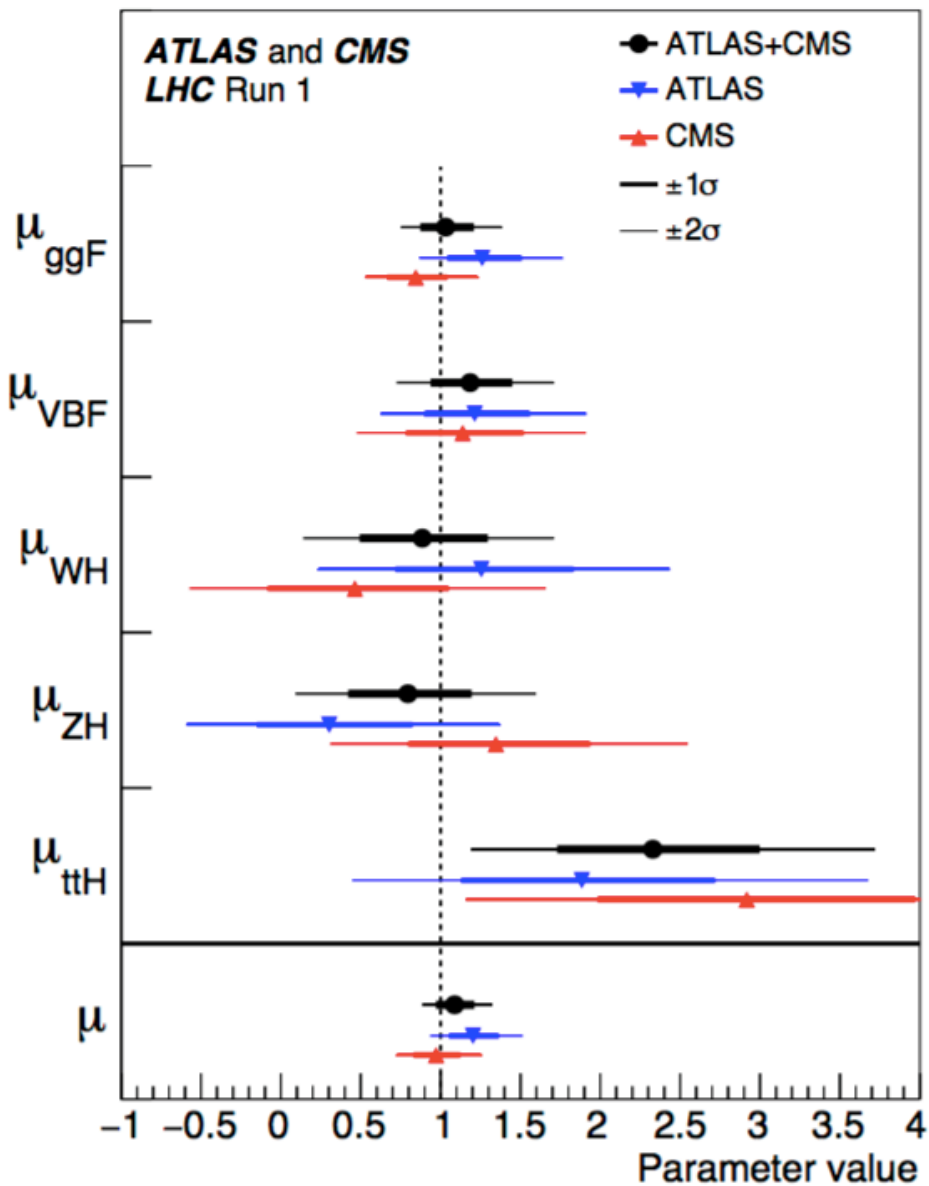
$$\gamma_f = \frac{2N_c}{(4\pi)^2} \int_0^\infty dp_E^2 p_E^2 \left(\frac{\Pi_{1Q}}{\Pi_Q} + \frac{\Pi_{1S}}{\Pi_S} + \frac{\Pi_{QS}^2}{p_E^2 \Pi_Q \Pi_S} \right),$$
$$\beta_f = \frac{N_c}{(4\pi)^2} \int_{\mu_f^2}^\infty dp_E^2 p_E^2 \left(\left(\frac{\Pi_{QS}^2}{p_E^2 \Pi_Q \Pi_S} + \frac{\Pi_{1Q}}{\Pi_Q} + \frac{\Pi_{1S}}{\Pi_S} \right)^2 - \frac{2(p_E^2 \Pi_{1Q} \Pi_{1S} - \Pi_{QS}^2)}{p_E^2 \Pi_Q \Pi_S} \right).$$

费米子贡献

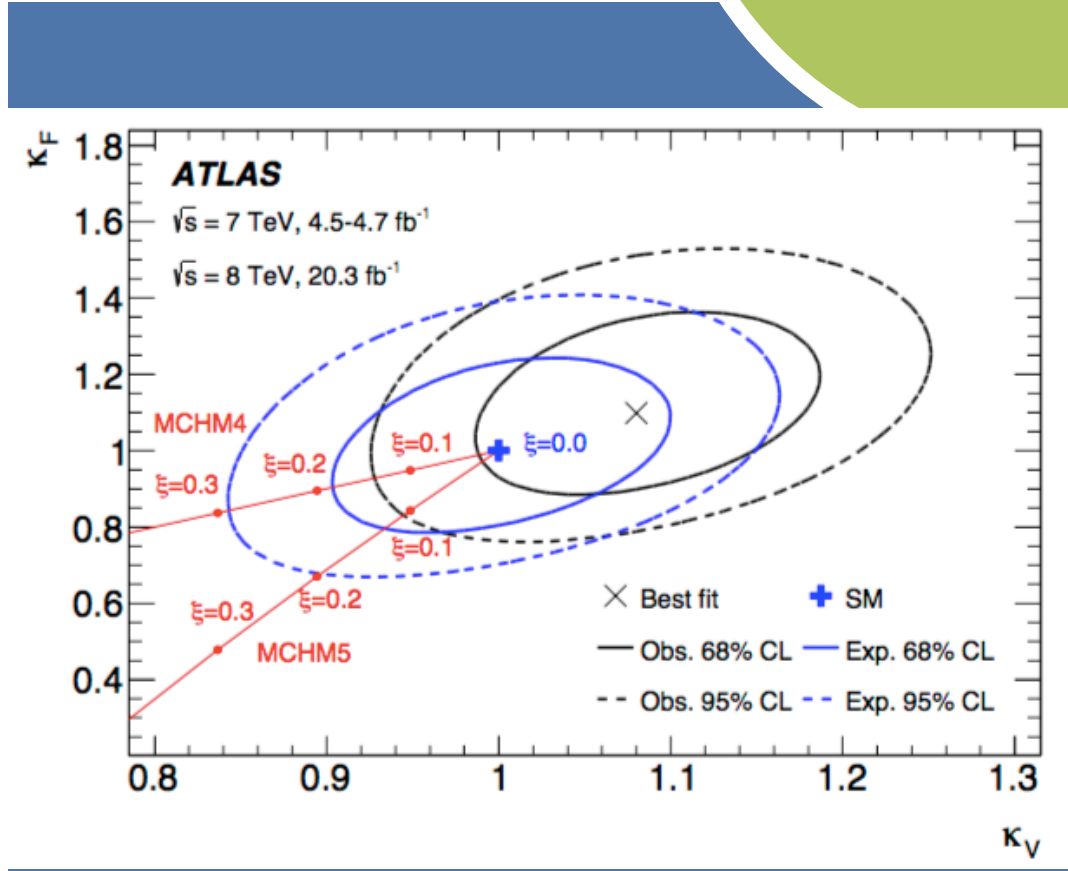
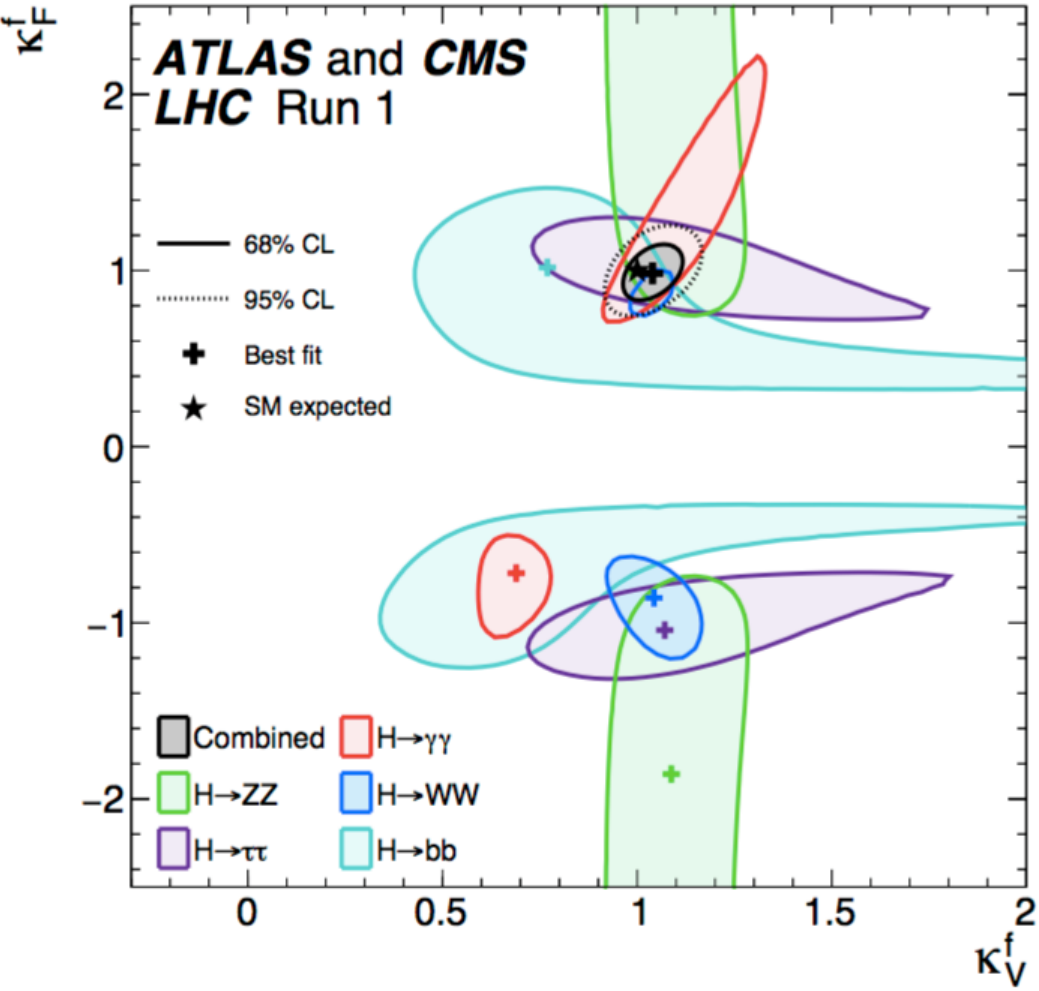
$$M_t^2(q^2, \langle h \rangle) = \frac{|\Pi_{t_L t_R}(q^2, \langle h \rangle)|}{\sqrt{\Pi_{t_L}(q^2, \langle h \rangle) \Pi_{t_R}(q^2, \langle h \rangle)}}.$$

Top 夸克质量

Higgs产生和衰变



Higgs物理



Top耦合为负的情况不再存在

Higgs 拟合 $\xi < 0.1$